December 19, 2002 M314 Final Exam Name : ____________________

Problem :	1	2	3	4	5
Points :	9	10	30	10	16
Score :					
Total :					

Instructions: Show all of your work; your work must justify your answers.

[1] Consider the systems of equations:

$$x + (k+5)y = h$$
$$x + y = 3$$

(a) Determine all values of k and h such that the system has no solutions. Explain your answer.

(b) Determine all values of k and h such that the system has infinitely many solutions. Explain your answer.

(c) Determine all values of k and h such that the system has exactly one solution. Explain your answer.

[2] Let A be an $n \times n$ matrix.

(a) If A^2 is the 0 matrix, prove that $\lambda = 0$ is an eigenvalue of A and that it is the only eigenvalue of A.

(b) If A^2 is the 0 matrix and A is diagonalizable, prove that A is the 0 matrix.

[3] In this problem, R_A is the reduced row echelon form of the matrix A. For each of the following statements, circle T if it is true and F if it is false. In addition, if it is false, write down a specific matrix A for which the statement does not hold.

(a) T F: For every matrix A, $\text{Nul}(A) = \text{Nul}(R_A)$.

(b) T F: For every matrix A, $\text{Row}(A) = \text{Row}(R_A)$.

(c) T F: For every matrix A, $\text{Col}(A) = \text{Col}(R_A)$.

(d) T F: Every $n \times n$ matrix A has the same characteristic polynomial as R_A.

(e) T F: For every $n \times n$ matrix A, $\det(A) = \det(R_A)$.

(f) T F: An $n \times n$ matrix A is invertible if and only if R_A is.

(g) T F: An $n \times n$ matrix A is diagonalizable if and only if R_A is.

(h) T F: For every matrix A, $\text{rank}(A) = \text{rank}(R_A)$.

(i) T F: For every matrix A, $\dim \text{Row}(A) = \dim \text{Row}(A^T)$.

(j) T F: For every matrix A, $\text{Nul}(A) = (\text{Row}(A))^\perp$.

[4] Let H be the subset $\{[a+1, b]^T : a, b \in \mathbf{R}\}$ of $\mathbf{R}^2$. Let W be the subset $\{[1, b]^T : b \in \mathbf{R}\}$ of $\mathbf{R}^2$.
(a) Is W a subspace of $\mathbf{R}^2$? Explain why or why not.

(b) Is H a subspace of $\mathbf{R}^2$? Explain why or why not.

[5] Let H be the span of the vectors $[0, 1, 0, 1]^T$, $[1, 0, 1, 0]^T$, $[0, 1, 1, 0]^T$, and $[1, 0, 0, 1]^T$. For each part, show your work or explain your answer.

(a) Find the dimension of H.

(b) Determine if $[1, 1, 1, 1]^T$ is in H.

(c) Find an orthogonal basis for H.

(d) Find $\text{proj}_H \mathbf{v}$ for $\mathbf{v} = [1, 2, 3, 4]^T$.