Harbourne M310 Spring 2006 Wednesday February 15, 2006

Exam 1: Name:____________________________

Instructions: Show all of your work and clearly explain your answers. No books or written notes are allowed during the exam. Do any 4 of the 5 problems. Use one page per problem. Be sure to clearly indicate which of the 5 problems you want graded. Each problem counts 25 points, for a total of 100.

[1] Let S be the set $\{a, b, c\}$.

(a) Define a relation on the set S by putting checkmarks in a labeled tic-tac-toe grid, as on the board. Check as many squares as possible, such that the relation you define is NOT reflexive.

(b) This time let the set S be the set of all people. Say person A is related to person B if A and B share a grandparent, but not a parent (i.e., if A and B are cousins but not siblings). For each of the properties reflexivity, symmetry and transitivity, determine whether or not the property holds for this relation. Justify your answer in each case.

[2] Prove the formula $1 + 3 + 5 + \cdots + (2n - 1) = n^2$ for each $n \geq 1$. (I.e., prove that the sum of the first n odd integers equals n^2.) Include each step of the proof in your answer.

[3] Let $a_1 = 3$, $a_2 = 5$, and define $a_{n+1} = 3a_n - a_{n-1}$ for $n \geq 2$. Prove that $a_k > 2^k$ for each integer $k \geq 1$.

[4]

(a) Use Euclid's method to find gcd(357, 918).

(b) Show how to use your work in (a) to find an integer solution to $918x + 357y = \gcd(357, 918)$.

(c) Find the least positive integer y such that $918x + 357y = \gcd(357, 918)$ has a solution where x also is an integer. Justify your answer.

[5]

(a) Explain why the least positive integer linear combination $111x + 74y$ of 111 and 74 is 37.

(b) Let k be an integer. Justify why $111x + 74y = k$ has a solution if and only if $37 | k$.