

[1] (22 points) An election is run. The candidates are Paul (P), Tom (T), Sally (S), and Ann (A). There are 17 voters. Here is a tabulation of their preference lists:

# Voters	5	4	4	2	2
First place	S	P	A	T	A
Second place	P	T	S	P	P
Third place	A	S	T	A	S
Fourth place	T	A	P	S	T

(a: 3 points) Determine the vote totals using plurality voting. Who is the winner?

Just count the number of first place votes for each candidate: S: 5 P: 4 A: 6 T: 2

Ann has the most votes so she wins.

(b: 2 points) Who wins if Sally drops out of the race?

If Sally drops out that means that Paul gets her first place votes now (look at the first preference list).

The vote totals are now: P: 9 A: 6 T: 2 Thus Paul now wins.

(c: 2 points) Do (a) and (b) give an example of a violation of a fairness criterion? If so which one? Explain.

The irrelevant alternatives criterion is violated: Sally (a loser) swung the election from Ann to Paul.

(d: 3 points) Determine the vote totals using the Borda count. Who is the winner?

P: 47 A: 42 S: 46 T: 35 Paul wins.

(e: 2 points) Does (d) give an example of a violation of a fairness criterion? If so which one? Explain.

No; although Paul wins the Borda count, while Ann won the plurality, this is not a violation of the majority Criterion, since Ann did not have a majority.

(f: 3 points) Indicate the order of elimination using plurality with elimination voting. Who wins?

T goes first, then S, then A leaving P the winner.

(g: 2 points) Suppose we switch P and A in the last column. Who now wins using plurality with elimination voting?

Again T goes first, but then A, and then P, leaving S the winner.

(h: 2 points) Does (g) give an example of a violation of a fairness criterion? If so which one? Explain.

The monotonicity criterion is violated, since the winner before moved up, but that made him lose.

(i: 3 points) Determine the vote totals using pairwise comparison voting. Who is the winner?

P: 2 S: 3 A: 1 T: 0 S is the winner.

[2] (8 points) Consider the weighted voting system $[20 | 13, 8, 7, 4]$.

(a) Which if any of the voters are dummies? Explain.

The voter of weight 4 is a dummy since no coalition ever needs those 4 votes to reach the quota; each coalition is either already at the quota or the 4 votes aren't enough.

(b) Which if any of the voters have veto power? Explain.

To have veto power the sum of the rest of the votes must be less than the quota.

Thus the voter with weight 13 is the only one with veto power.

(c) Which if any of the voters are dictators? Explain.

There are no dictators. A dictator weight must be as big as or bigger than the quota.

(d) What is the Banzhaf power index of each voter?

There are only three winning coalitions: $\{A,B,C\}$, $\{A,B\}$, $\{A,C\}$.

Each member of each coalition is critical (if you take any voter away, that coalition does not have enough votes to win). Thus A is critical 3 times and B and C just twice.

The Banzhaf indices are: $3/7$ for A, and $2/7$ for B and C.