

[1] (6 points) John and Jane get stuck watching the baby. They are told to watch the baby for a total of 8 hours (4 hours in the morning and 4 in the afternoon). They agree to use the two player divide and choose method to split babysitting hours between them. In order to keep his morning hours down, John is willing to do 2 afternoon hours for each morning hour. For Jane, the hours are all the same. Assume John is the divider. If he were to create two schedules, both with 2 morning hours and 2 afternoon hours, he realizes he'd be stuck doing 2 morning hours in addition to 2 afternoon hours no matter what. Knowing that Jane doesn't care so much when she does her hours, how might John divide the hours into two schedules that he would see as having equal value but which would minimize his morning shift? Explain what you think John would do (and why), and how you think Jane would choose.

John would make two schedules: one with 3 morning hours, the other with 4 afternoon hours and a morning hour. These have the same value to John, since if we subtract from each schedule one morning hour, we're left with 4 afternoon hours versus 2 morning hours. Jane would take the 3 morning hours, since the other shift is longer.

[2] (6 points) Judy, Sam and Terri agree to divide a carton of Neapolitan ice cream (which has 12 ounces each of chocolate, vanilla and strawberry). They agree to use the three person lone chooser method, in which Judy divides the carton into two portions of equal value to her. Then Sam picks one of these two and gives the other to Judy. Finally, Sam and Judy each divide their portions into three pieces of what they see as having equal value, at which point Terri picks one portion each from Judy and Sam.

Assume Judy hates chocolate but likes vanilla and strawberry the same. She decides therefore to make one portion with 6 oz chocolate and 12 oz vanilla, with the other portion being 6 oz chocolate and 12 oz strawberry. Thus no matter what Sam picks, Judy can get stuck with at most half of the chocolate. Assume Sam loves strawberry, vanilla and chocolate equally. Sam decides to leave Judy the vanilla-chocolate portion and keep the strawberry-chocolate portion. Judy now cuts her portion into three pieces. She cuts the vanilla into three equal pieces and puts the chocolate with one of those pieces. Thus the three pieces are: a 4 oz portion of vanilla, another 4 oz portion of vanilla, and a third portion which is 4 oz vanilla plus 6 oz chocolate. Sam divides his portion into: two 6 oz portions of strawberry and one 6 oz portion of chocolate.

(a) Assume Terri loves strawberry and chocolate, but doesn't like vanilla so much. What choices might Terri make?

Terri would take the portion consisting of 6 oz chocolate plus 4 oz vanilla from Judy, and any one of the three portions from Sam.

(b) Is the end result envy free, or does someone prefer what someone else got? Indicate each person who prefers what someone else got. Explain your answer.

Sam prefers Terri's portion, since Terri ends up with more ice cream than Sam and Sam likes all of the flavors. The rest of your answer depends on what you said Terri did. If Terri picks a strawberry portion from Sam, then Judy will envy Terri since Terri will have more of the flavors Judy likes. If Terri picks the chocolate portion from Sam, then Judy will envy Sam because Sam will have more of the flavors Judy likes.

[3] (14 points) A local charity is giving away a 12 pack of root beer for a donation of \$20. Bobby, Mikey and Janey pool their money to make a \$20 donation. Bobby contributes \$8, Janey \$7, and Mikey \$5.

(a) How should the cans be apportioned between them if each child's apportionment is proportional to that child's contribution, assuming that Hamilton's method is used to deal with fractional parts of cans? Fill in the table below to show your answer.

Child	Standard Quota	Hamiltonian Apportionment
Bobby	$12(8/20) = 4.8$	5
Janey	$12(7/20) = 4.2$	4
Mikey	$12(5/20) = 3$	3

(b) How should the cans be apportioned between them if each child's apportionment is proportional to that child's contribution, assuming that Jefferson's method is used to deal with fractional parts of cans? Fill in the table below to show your answer, and indicate the standard divisor D and the modified divisor d which you are using.

Standard divisor $D = \frac{20}{12} = 5/3 = 1.667$

Note the roundings down of the standard quotas add up only to 11 so we need to reduce the standard divisor to get our modified divisor. I'll try $d = 1.5$ as a first guess for the modified divisor d. That turns out to work.

Child	Standard Quota	Modified Quota	Jeffersonian Apportionment
Bobby	4.8	$8/1.5 = 5.333$	5
Janey	4.2	$7/1.5 = 4.667$	4
Mikey	3	$5/1.5 = 3.333$	3

[4] (4 points) A school district assigned 35 instructional assistants to five schools based on enrollment figures. The budget allowed for the hiring of 2 additional assistants. Consider the following apportionment numbers before and after the increase in instructional assistants. Is this an example of a paradox? If so, which paradox has occurred? Explain.

School	Original Apportionment	New Apportionment
Cascades	9	9
Seven Oaks	11	10
Riverview	6	7
Pioneer	4	5
Hamilton Creek	5	6

This is an example of an increase in the total number of items to be apportioned causing a reduction in someone's apportionment; i.e., it is an example of the Alabama paradox.