

Practice Quiz 5 Solutions: on Chapter 5

[1] (20 points) Suppose Gotham City has a 5 seat city council for its 3 districts and the 5 seats are apportioned by population to the 3 districts.

(a) Suppose that the standard quotas are as given in the following table. Fill in the table. Explain how you obtained the apportionment in each case.

District	Standard Quotas	Hamiltonian Apportionment	Lowndes' Apportionment
Shadyside	1.3	1	2
Squirrel Hill	1.2	1	1
Oakland	2.5	3	2

Hamilton's apportionment gives each district the round down of the standard quota, then assigns the remaining seats according to which standard quotas have biggest fractional part. Oakland's is biggest, so they get the leftover seat.

Lowndes' Apportionment gives each district the round down of the standard quota, then assigns the remaining seats according to which round downs represent the smallest percentages of the standard quotas. The fractions are:

Shadyside  $77\% = 1/1.3$       Squirrel Hill  $83\% = 1/1.2$       Oakland  $80\% = 2/2.5$

Thus Shadyside's percentage is least (i.e., if they were to just get the round down, they would be shorted the most, by percentage) so they get the leftover seat.

(b) Suppose that the modified quotas are as given in the following table. Answer the question and then fill in the table, as appropriate.

(i) If you were using Jefferson's apportionment method and the modified quotas were as shown, can you now fill in the table? If so do so. If not, answer the following two questions:

- (\*) recompute the modified divisors using a bigger modified divisor False
- (\*) recompute the modified divisors using a smaller modified divisor False

District	Modified Quotas	Jeffersonian Apportionment
Shadyside	1.57	1
Squirrel Hill	1.45	1
Oakland	3.05	3

(ii) If you were using Webster's apportionment method and the modified quotas were as shown, can you now fill in the table? If so do so. If not, answer the following two questions:

- (\*) recompute the modified divisors using a bigger modified divisor True
- (\*) recompute the modified divisors using a smaller modified divisor False

District	Modified Quotas	Webster's Apportionment
Shadyside	1.57	Not Applicable
Squirrel Hill	1.45	Not Applicable
Oakland	3.05	Not Applicable

[2] (10 points) A local charity is giving away a 12 pack of root beer for a donation of \$20. Bobby, Mikey and Janey pool their money to make a \$20 donation. Bobby contributes \$8, Janey \$7, and Mikey \$5.

(a) How should the cans be apportioned between them if each child's apportionment is proportional to that child's contribution, assuming that Hamilton's method is used to deal with fractional parts of cans? Fill in the table below to show your answer.

Child	Standard Quota	Hamiltonian Apportionment
Bobby	$12(8/20) = 4.8$	5
Janey	$12(7/20) = 4.2$	4
Mikey	$12(5/20) = 3$	3

(b) How should the cans be apportioned between them if each child's apportionment is proportional to that child's contribution, assuming that Jefferson's method is used to deal with fractional parts of cans? Fill in the table below to show your answer, and indicate the standard divisor D and the modified divisor d which you are using.

Standard divisor  $D = \frac{20}{12} = 5/3 = 1.667$

Note the roundings down of the standard quotas add up only to 11 so we need to reduce the standard divisor to get our modified divisor. I'll try  $d = 1.5$  as a first guess for the modified divisor d. That turns out to work.

Child	Standard Quota	Modified Quota	Jeffersonian Apportionment
Bobby	4.8	$8/1.5 = 5.333$	5
Janey	4.2	$7/1.5 = 4.667$	4
Mikey	3	$5/1.5 = 3.333$	3

[3] (10 points) A school district assigned 35 instructional assistants to five schools based on enrollment figures. The budget allowed for the hiring of 2 additional assistants. Consider the following apportionment numbers before and after the increase in instructional assistants. Is this an example of a paradox? If so, which paradox has occurred? Explain.

School	Original Apportionment	New Apportionment
Cascades	9	9
Seven Oaks	11	10
Riverview	6	7
Pioneer	4	5
Hamilton Creek	5	6

This is an example of an increase in the total number of items to be apportioned causing a reduction in someone's apportionment; i.e., it is an example of the Alabama paradox.