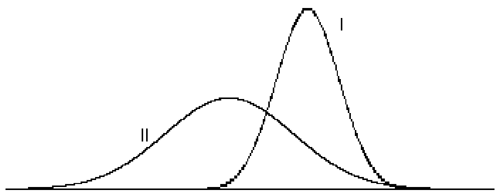
Practice Quiz 3 covering Chapter 11 Solutions

[1] Sketch two normal distributions using the same x-axis. Label them distribution I and distribution II, such that distribution I has a larger mean but a smaller standard distribution.



Note that distribution I is taller but skinnier (skinnier since it has the smaller standard deviation) and its peak is further to the right (since it has the larger mean).

[2] Using Table 11.3 on p. 709, determine what percentage of a normally distributed population has z values in the range $-1.5 \le z \le 0.5$.

The percentage from -1.5 to 0 is 43.32%. The percentage from 0 to 0.5 is 19.15%, so the total percentage is 43.32 + 19.15 = 62.47%.

[3] Assume IQ scores for adults are normally distributed with a mean of 100 and a standard deviation of 15.
(a) What percentage of adults have IQ scores in the range 70 to 130? Explain how you obtain your answer.

Since 70 to 130 is the range form 2 standard deviations below the mean to 2 standard deviations above the mean, we know that about 95% of the population s in this range, using the 68-95-99.7 rule.

(b) What IQ score must one have in order for 90.32% of the population to have that score or less? (Hint: use Table 11.3 on p. 709.)

From Table 11.3 we see that in the range from 0 to a z value of z = 1.3, the percentage is 40.32%. Add to that the 50% of the population with negative z values gives 90.32% of the population having a z value of 1.3 or less. The x value (i.e., the IQ score) corresponding to z = 1.3 is x = 100 + 1.3(15) = 100 + 19.5 = 119.5. Thus 90.32% of adults have IQ scores of 119.5 or less.

[4] (a) Find the 95% confidence interval for a survey on taxes which used a sample size of 2500 and found a sample proportion of p^ = 10% in favor of higher taxes. Explain how you obtain your answer.

The standard error (i.e., the estimated standard deviation for the sampling distribution) is s^ = √(p^(1–p^)/n) = √(.1*.9/2500) = 0.006 = 0.6%. Thus the 95% confidence is p^ ± 2s^ hence 10% ± 1.2% or 8.8% to 11.2%.

(b) What sample size n is needed in order for the sampling distribution for samples of that sample size to be approximately normally distributed, if the population proportion p is 10%? Explain how you obtain your answer.

As discussed on p. 731 of the book, what we need is for both of the conditions p – 3√(p(1-p)/n) > 0 and p + 3√(p(1-p)/n) < 1 to hold. Solving each condition for n gives n > 9(1–p)/p and n > 9p/(1–p). Thus we need n > 9(0.9)/0.1 = 81 and n > 9(0.1)/0.9 = 1. Hence we need n to be at least 82.

(c) What sample size n is needed in order both for the sampling distribution to be approximately normal and for at least 95% of all possible samples of that sample size to have a sample proportion p^ in the range 10% ± 2%, if the population proportion p is 10%? Explain how you obtain your answer.

We saw from part (b) that we need a sample size of at least 82 for the distribution to be close to normal, so whatever our final answer ends up being, it needs to be at least 82. Since the 95% margin of error is 2%, we know the standard deviation of the sampling distribution is half that, or 1%. The formula for the standard deviation is √(p(1–p)/n) so we have √(p(1–p)/n) = 1% = 0.01. Now we just solve for n. Solving for n gives the formula $n = p(1–p)/(0.01^2)$ and plugging in gives $n = 0.10(0.90)/(0.01^2) = 900$. Since n = 900 is more than 82, our final answer is that we need a sample size of at least 900. This will ensure both that the distribution is close to being normal and that a random sample of that sample size has at least a 95% chance of having a sample proportion p^ that is within 2 percentage points of the actual population proportion p.

[5] In reference to a news article (saying that a national survey of 1000 randomly chosen respondents has found that national sentiment on some topic is 89% ± 2% in support of the issue), a colleague of yours mentions to you that journalists just don't understand that a sample of 1000 individuals chosen at random from a population of 100,000,000 is too small a sample to reliably "show" or "find" the feelings of that large mass. You decide (possibly against your better instincts) to explain to your colleague in what sense it is reliable, and how reliable it is. What do you say?

When dealing with large populations, the level of reliability of a sample depends only on the size of the sample; the size of the population doesn't matter. There is a 95% chance that the sample measurement showing 89% support is accurate to within 2 percentage points, if the sample size was 1000. It doesn't show or find anything for sure, but no measurements of anything ever do. There is always some chance of error. The sample measurement shows (with a reliability of 95%) that national sentiment is within 2 percentage points of 89%.