Practice Quiz 1 covering Chapters 1 and 3 Solutions

[1] (8 pts) Suppose the UPC code on a certain box of cereal is 0 21000 x1061 7. What should the digit x be? Show how you determine the answer.

\[
3(0 + 1 + 0 + x + 0 + 1) + 2 + 0 + 0 + 1 + 6 + 7 = 3x + 22\]

To get an even multiple of 10 we need 3x = 6.

[2] (8 pts) The Postnet bar code shown here has a single error in which either one vertical bar which should be long is short or vice versa:

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...|| |...|||...|||.|.||......|||.|...|..|||......||..|.|
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Show how you fix the error, then express the correct zip code in ordinary characters.

1 7 x 0 1 9 4 0 1 2

The sum is x + 25 so to get an even multiple of 10 we need x = 5. Thus the correct code is: 17501-9401.

[3] (15 pts) An election is run. The candidates are Paul (P), Tom (T), Sally (S), and Ann (A). There are 17 voters. Here is a tabulation of their preference lists:

<table>
<thead>
<tr>
<th># Voters</th>
<th>5</th>
<th>4</th>
<th>4</th>
<th>2</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>First place</td>
<td>T</td>
<td>A</td>
<td>S</td>
<td>P</td>
<td>S</td>
</tr>
<tr>
<td>Second place</td>
<td>A</td>
<td>P</td>
<td>T</td>
<td>A</td>
<td>A</td>
</tr>
<tr>
<td>Third place</td>
<td>S</td>
<td>T</td>
<td>P</td>
<td>S</td>
<td>T</td>
</tr>
<tr>
<td>Fourth place</td>
<td>P</td>
<td>S</td>
<td>A</td>
<td>T</td>
<td>P</td>
</tr>
</tbody>
</table>

(a) Determine the vote totals using plurality voting. Who is the winner? Totals: A: 4 P: 2 S: 6 T: 5 Winner: S

(b) Assume Sally drops out of the race but that the preference list above remains otherwise the same. State the vote totals in this situation for each candidate using plurality voting. Who is the winner now?

Totals: A: 6 P: 2 T: 9 Winner: T

(c) Determine the vote totals using the Borda count with the original preference list given above. Who is the winner?

Totals: A: 47 P: 35 S: 42 T: 46 Winner: A

(d) Does (c) give an example of a violation of the Majority Criterion? Circle your answer: YES NO

Explain: For a violation of the Majority Criterion, there must have been a candidate who had a majority of the first place votes but who lost the election. In (a) no one had a majority of first place votes.

(e) Indicate the order of elimination using plurality with elimination voting. Who wins?

First candidate eliminated: P Second candidate eliminated: T Third candidate eliminated: S Winner: A

[4] (9 pts) Consider the weighted voting system [13 | 12, 7, 1]. Let A, B and C be the players, where A has 12 votes, B has 7, and C has 1.

(a) Which if any of the voters are dummies? Explain. There are no dummies; if anyone would be a dummy it would be C, but A needs C's vote if he can't get B to vote his way.

(b) Which if any of the voters have veto power? Explain. Player A has veto power; a measure cannot pass without the support of A, but neither of the others has a veto, since even if B votes NO, a measure can pass if both A and C vote YES.

(c) Which if any of the voters are dictators? Explain. No one is a dictator, since no one alone has a quota.

(d) What is the Banzhaf power index of each voter? There are three winning coalitions, \(\{A^*, B^*\}, \{A^*, C^*\}\) and \(\{A^*, B, C\}\); the asterisks indicate which players are critical in each winning coalition. Thus the Banzhaf index for A is 3/5 or 60%, while for B and C it is 1/5 or 20% each.