Practice Quiz 1   Solutions

[1] (6 points) Suppose the UPC code on a certain box of cereal is 0 21051 x2061 6. What should the digit x be? Show how you determine the answer. (Remember: the UPC code is such that when you add every other digit starting with the first, triple the result and then add the remaining digits, you get an even multiple of 10.)

3(0 +   1   +   5   +   x   +   0   +   1) +
      2   +   0   +   1   +   2   +   6   +   6   = 21 + 3x + 17 = 38 + 3x so to get an even multiple of 10 we need x = 4.

[2] (6 points) The Postnet bar code shown here has a single error in which either one vertical bar which should be long is short or vice versa:

|.|.|..||..|..||..|....|||.|...|..||.....||....|.||

Show how you fix the error, then express the correct zip code in ordinary characters.

.|.|.   ..||.   .|..| |..|.   ...|| |.|..   .|..| |....   .||..   ..|.|
  5   3   4   8   1   9   4   x   6   2

The sum is x + 42 so to get an even multiple of 10 we need x = 8.
Thus the correct zip code is: 53481-9486.

[3] (15 points) An election is run. The candidates are Paul (P), Tom (T), Sally (S), and Ann (A). There are 17 voters. Here is a tabulation of their preference lists:

# Voters	5	5	4	1	2
First place	S	T	P	A	P
Second place	T	A	S	T	T
Third place	P	S	A	P	S
Fourth place	A	P	T	S	A

(a) Determine the vote totals using plurality voting. Who is the winner? Totals: A: 1 P: 6 S: 5 T: 5 Winner: P
(b) Assume Sally drops out of the race but that the preference list above remains otherwise the same.
State the vote totals in this situation for each candidate using plurality voting. Who is the winner now? Totals: A: 1 P: 6 T: 10   Winner: T
(c) Do (a) and (b) give an example of a violation of a fairness criterion? If so, which one?
Explain: Yes, the Irrelevant Alternatives Criterion is violated, since Sally, a loser, dropping out has resulted in a different winner.
(d) Determine the vote totals using the Borda count with the original preference list given above. Who is the winner?

Totals: A: 34   P: 41   S: 47   T: 48   Winner: T

	1st	2nd	3rd	4th	Points
A	1	5	4	7	34
P	6	0	6	5	41
S	5	4	7	1	47
T	5	8	0	4	48

(e) Indicate the order of elimination using plurality with elimination voting, given the original preference list given above. Who wins?
First candidate eliminated: A Second candidate eliminated: S Third candidate eliminated: P   Winner: T
round 1:   A: 1 P: 6 S: 5 T: 5
round 2:   P: 6 S: 5 T: 6
round 3:   P: 6 T: 11
Winner: T

[4] (8 points) Consider the weighted voting system [19 : 12, 7, 1]. Let A, B and C be the players, where A has 12 votes, B has 7 and C has 1.
(a) Which if any of the voters are dummies? Explain. Player C is a dummy. There are only two winning coalitions, {A,B,C} and {A,B}, and neither one needs C's votes to remain winning. However, players A and B are both needed in, for example, {A,B} for it to remain winning, so neither is a dummy.
(b) Which if any of the voters have veto power? Explain. Players A and B have veto power, since each is in every winning coalition. Since player C is a dummy, C does not have veto power.
(c) Which if any of the voters are dictators? Explain. There is no dictator, since no-one has enough votes to pass a measure by himself.
(d) What is the Banzhaf power index of each voter?
The winning coalitions are: {A,B,C} and {A,B}. In both, A and B are critical, so the Banzhaf power indices are: A: 2/4 B: 2/4 C: 0/4

[5] (5 points) If all of the fans in attendance at the next home football game were to flood the field after the game, is it conceivable that they could all fit on the field? (Attendance at football games last year averaged about 86000. A football field is 100 yards by 50 yards.)
A football field covers 50*100 = 5000 square yards, and each square yard covers 3*3 = 9 square feet so a football field covers 50*100*9 = 45000 square feet. Dividing the 45000 square foot area of the field by the 86000 people at the game gives a crowd density of 45000/86000 = 0.52 square feet per person, so each person would have less space than the area of an 8.5 by 11 piece of paper. Said another way, the crowd would be about 5 times as dense as what is usually regarded as a dense crowd. Thus it is unlikely they could all fit on the field.