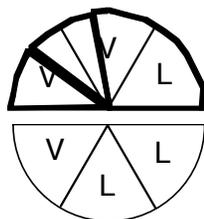


Fair Division Practice Quiz Solutions:

[1] Continuous division problem by lone chooser method. Ann, Bob and Cindy decide to split a \$36 cake between them. The cake is half vanilla and half lemon. Ann will divide the cake into what she regards as two pieces of equal value. Bob will pick one of the two pieces and Ann gets the piece Bob did not pick. Then Ann will divide her piece into three pieces of what she regards as equal value. Bob will split his piece into three pieces of what he regards as equal value. Then Cindy gets to pick one of Ann's three pieces and one of Bob's three pieces. Thus everybody ends up with two pieces of cake.



Ann likes vanilla and lemon equally and splits the cake as shown (one piece is two thirds vanilla and one third lemon, and the other piece is two thirds lemon and one third vanilla).

Assume Bob loves vanilla but hates lemon, while Cindy loves lemon but hates vanilla.

(a) Which of the two pieces shown will Bob pick?

Bob will pick the top half since it is two thirds vanilla, whereas the bottom half is only one third vanilla.

(b) For simplicity, assume Bob divides his piece into three contiguous sectors. How would Bob divide his piece in that case?

Bob would divide the 120° of vanilla into three pieces of 40° each, and toss in the lemon wedge with one of these 40° vanilla wedges, as shown in bold.

(c) Assume Ann divides her piece into three contiguous sectors. How would she do it?

She would divide them just as shown: a 60° vanilla wedge and two 60° lemon wedges.

(d) Which piece would Cindy pick from Ann and which piece from Bob?

Cindy would pick a lemon wedge from Ann, and the portion from Bob that had the lemon wedge included with a 40° vanilla wedge.

(e) Fill in the table showing how much each of the three sees their three portions are worth. Indicate who envies whom.

Ann's values: each wedge is worth \$6.

Bob's values: each vanilla wedge is worth \$12, but the lemon wedges are worth \$0.

Cindy's values: each lemon wedge is worth \$12, the vanilla wedges are worth \$0.

	Ann's portion	Bob's portion	Cindy's portion
Values as Ann sees them	\$6 + \$6 = \$12	\$6 + 1/3*\$6 = \$8	\$6 + \$6 + \$4 = \$16
Values as Bob sees them	\$12 + \$0 = \$12	\$12 + 1/3*\$12 = \$16	(2/3)\$12 + 0 = \$8
Values as Cindy sees them	\$0 + \$12 = \$12	\$0	\$12 + \$12 = \$24

Ann envies Cindy, but neither Bob nor Cindy envy anyone else.

[2] Repeat the problem using the lone divider method, assuming Bob is the lone divider. Assume Bob divides the cake into 3 pieces as follows: one piece is one of the vanilla wedges, another piece is another of the vanilla wedges, and the third piece is the third vanilla wedge plus all of the lemon. Let Ann be the first chooser and let Cindy be the second chooser. Explain what Ann and Cindy would do, what portions everyone gets and determine the value each person assigns to everyone's portions and whether anyone envies anyone else.

As before, Ann likes vanilla and lemon equally, Bob loves vanilla but hates lemon, and Cindy loves lemon but hates vanilla.

(a) Which pieces are acceptable to Ann? Which are acceptable to Cindy?

For a piece to be acceptable, it must be worth at least \$12. But the vanilla wedges are worth

only \$6 to Ann, so only the combined vanilla-lemon wedge (at \$24) is acceptable to her. And the vanilla wedges are worth nothing to Cindy, so again only the combined vanilla-lemon wedge (at \$36) is acceptable to her.

(b) Explain what portions Ann and Cindy get, and what Bob gets.

Since only piece is acceptable to Ann and Cindy, and it's the same piece, Bob gets to pick either of the other two pieces. They're the same, a vanilla wedge, so he picks one. Then Ann and Cindy put the other vanilla wedge together with the third vanilla wedge and the lemon wedges. Ann, being first chooser, divides that into two equal portions, one portion has 2 of the lemon wedges and half of the third lemon wedge, and the other portion has the other half of the lemon wedge plus two vanilla wedges. Cindy, as second chooser, makes her choice: she'd choose the portion with 2.5 lemon wedges. Ann gets what's left (i.e., the 2 vanilla wedges and half of a lemon wedge).

(c) Fill in the table showing how much each of the three sees their three portions are worth. Indicate who envies whom.

	Ann's portion	Bob's portion	Cindy's portion
Values as Ann sees them	\$6 + \$6 + \$3 = \$15	\$6	\$6 + \$6 + \$3 = \$15
Values as Bob sees them	\$12 + \$12 + \$0 = \$24	\$12	\$0
Values as Cindy sees them	\$0 + \$0 + \$6 = \$6	\$0	\$12 + \$12 + \$6 = \$30

Bob envies Ann. Nobody else envies anybody.

[3] Discrete division problem (sealed bids method):

Judy, Cathy and Sharon want to divide among them a piano and a lamp. Here is how they bid:

	Judy	Cathy	Sharon
piano	\$9000	\$3600	\$4500
lamp	\$360	\$900	\$450

Determine who got which item and how much money each person got.

First do the piano:

Judy gets the piano and puts \$9000 into the pot. She then withdraws \$9000/3 = \$3000, Cathy withdraws \$3600/3 = \$1200, and Sharon withdraws \$4500/3 = \$1500. This leaves \$9000 - \$3000 - \$1200 - \$1500 = \$3300, which is divided equally between them (at \$1100 per person).

Now do the lamp:

Cathy gets the lamp and puts \$900 into the pot. Then Cathy withdraws her third (\$300), Judy her third (\$120) and Sharon her third (\$150), leaving \$330 which is split evenly between them (at \$110 per person).

Thus Judy gets the piano but paid out \$9000 - (\$4100+\$230) = \$9000 - \$4330 = \$4670, Cathy gets the lamp and \$2300 + \$410 - \$900 = \$2710 - \$900 = \$1810, and Sharon gets \$2600 + \$260 = \$2860.

Note that Judy sees herself as getting a total value of \$9000 - \$4670 = \$4330, which is more than one third of the \$9360 value that she put on the lamp and piano. She feels that the value of what Cathy got is \$1810+\$360=\$2170, and that the value of what Sharon got is \$2600. Cathy's values for what everyone got are: \$3600-\$4670 = -\$1070 for Judy; \$2710 for herself (which is more than her fair share of one third of the \$3500 value of the lamp and piano); and \$2600 for Sharon. Finally, Sharon sees Judy as having gotten a value of -\$170, Cathy as having gotten \$2260 and herself as having gotten \$2600 (which again is more than one third the \$4950 total value of the lamp and piano, as she sees it). So in this case everyone feels that no-one else got a deal as good, and all of the participants feel they got at least a fair share.