

Instructions: Answer each question, and explain your answer. An answer alone is not enough for full credit. Your explanation must be clear and show how to get the answer. Do not put your answers on the quiz handout; use additional sheets of paper. This is an open book quiz.

[1] (5 points) The UPC code on a can of Alpo dog food is 0 11132 00361 "?". What should the check digit be? Show how you determine the answer. (Remember: the UPC code is such that when you add every other digit starting with the first, triple the result and then add the remaining digits, you get an even multiple of 10.)

$$3(0 + 1 + 3 + 0 + 3 + 1) + (1 + 1 + 2 + 0 + 6 + x) = 24 + 10 + x = 34 + x$$

For this to be a multiple of 10 we need $x = 6$.

[2] (8 points) The Postnet bar code shown here has a single error in which either one vertical bar which should be long is short or vice versa:

||.....|.|...||...||.|.|...|.|...|.....|.....|

Show how you fix the error, then express the correct zip code in ordinary characters. (Remember: the outside bars are framing bars which you ignore; also, the digit Postnet bar codes are as follows:

1: ..|| 2: ..|| 3: ..|| 4: ..|| 5: ..|| 6: ..|| 7: ..|| 8: ..|| 9: ..|| 0: ..||

Moreover, the digits in a valid Postnet code must sum to an even multiple of 10.)

||... ..|| ..|| ..|| ..|| ..|| ..|| ..|| ..|| ..||
0 2 1 3 9 4 x 0 7 1

The sum of the digits is $27 + x$, so to get a multiple of 10 we need x to be 3.

[3] (9 points) An election is run. The candidates are Paul (P), Tom (T), Sally (S), and Ann (A). There are 17 voters. Here is a tabulation of their preference lists:

# Voters	5	4	4	2	2
First place	S	P	A	T	A
Second place	P	T	S	P	P
Third place	A	S	T	A	S
Fourth place	T	A	P	S	T

(a) Determine the vote totals using plurality voting. Who is the winner?

Just count the number of first place votes for each candidate: S: 5 P: 4 A: 6 T: 2

Ann has the most votes so she wins.

(b) Who wins if Sally drops out of the race?

If Sally drops out that means that Paul gets her first place votes now (look at the first column in the table).

The vote totals are now: P: 9 A: 6 T: 2 Thus Paul now wins.

(c) Do (a) and (b) give an example of a violation of a fairness criterion? If so which one? Explain.

The irrelevant alternatives criterion is violated: Sally (a loser) swung the election from Ann to Paul.

(d) Determine the vote totals using the Borda count. Who is the winner?

P: 47 A: 42 S: 46 T: 35 Paul wins.

(e) Does (d) give an example of a violation of a fairness criterion? If so which one? Explain.

No; although Paul wins the Borda count, while Ann won the plurality, this is not a violation of the majority Criterion, since Ann did not have a majority.

(f) Indicate the order of elimination using plurality with elimination voting. Who wins?

T goes first, then S, then A leaving P the winner.

(g) Suppose we switch P and A in the last column. Who now wins using plurality with elimination voting?

Again T goes first, but then A, and then P, leaving S the winner.

(h) Does (g) give an example of a violation of a fairness criterion? If so which one? Explain.

The monotonicity criterion is violated, since the winner before moved up, but that made him lose.

(i) Determine the vote totals using pairwise comparison voting. Who is the winner?

P: 2 S: 3 A: 1 T: 0 S is the winner.

[4] (8 points) Consider the weighted voting system [20 | 13, 8, 7, 4].

(a) Which if any of the voters are dummies? Explain.

The voter of weight 4 is a dummy since no coalition ever needs those 4 votes to reach the quota; each coalition is either already at the quota or the 4 votes aren't enough.

(b) Which if any of the voters have veto power? Explain.

To have veto power the sum of the rest of the votes must be less than the quota.

Thus the voter with weight 13 is the only one with veto power.

(c) Which if any of the voters are dictators? Explain.

There are no dictators. A dictator's weight must be as big as or bigger than the quota.

(d) What is the Banzhaf power index of each voter? Let A, B, C and D be the players, where A has 13 votes, B has 8, etc.

There are six winning coalitions. Here is each one followed by which players are critical:

{A,B,C,D}: A; {A,B,C}: A; {A,B,D}: A, B; {A,C,D}: A, C; {A,B}: A, B; {A,C}: A, C.

Thus A is critical 6 times, B and C 2 times each, and D never, so $6+2+2 = 10$.

Thus the Banzhaf indices are: 6/10 for A, 2/10 for B and C, and 0/10 for D.