1:

(a) First, by comparison with a p-series with $p < 1$,

$$\sum_{k=1}^{\infty} \frac{3k^3 - 1}{k^{4.07} + 2} \geq \sum_{k=1}^{\infty} \frac{k^3}{2k^{4.07}} = (1/2) \sum_{k=1}^{\infty} \frac{1}{k^{0.93}}$$

diverges, so

$$\sum_{k=1}^{\infty} (-1)^k \frac{3k^3 - 1}{k^{4.07} + 2}$$

does not converge absolutely. But the series is an alternating series; by looking at the graph of $\frac{x^3}{2x^{4.07}}$ we see the absolute values of the terms are decreasing, and since their limit is 0, the alternating series test tell us the series converges. Thus the series is conditionally convergent.

(b) The series

$$\sum_{k=1}^{\infty} (-1)^k \left(\frac{4k - 1}{3k + 2} \right)^k$$

diverges by the n-th term test (the terms do not have limit 0).

(c) The series

$$\sum_{k=1}^{\infty} \frac{99^k k!}{(2k + 1)!} \leq \sum_{k=1}^{\infty} \frac{99^k}{(2k + 1) \cdots (k + 2)(k + 1)} \leq \sum_{k=1}^{\infty} \frac{99^k}{(k + 1)^k} =$$

$$\sum_{k=1}^{\infty} (99/(k + 1))^k \leq \sum_{k=1}^{98} (99/k + 1)^k + \sum_{k=99}^{\infty} (99/100)^k$$

converges by the comparison test (since $\sum_{k=1}^{98} (99/k + 1)^k$ is a finite sum and the geometric series $\sum_{k=99}^{\infty} (99/100)^k$ converges), hence the series

$$\sum_{k=1}^{\infty} (-1)^k \frac{99^k k!}{(2k + 1)!}$$

converges absolutely.

2: Use the ratio test: the limit as k goes to ∞ of

$$\left| \frac{\frac{(x-2)^{k+2}}{k+1}}{\frac{(x-2)^{k+1}}{k}} \right|$$

is $|x - 2|$, so the series converges for $|x - 2| < 1$ and diverges for $|x - 2| > 1$. Thus the radius of convergence is 1, centered at $x = 2$. When $x = 1$, we get the harmonic series times -1, which diverges. When $x = 3$, we get an alternating series, whose terms decrease in absolute value with limit 0, hence is convergent. Thus the interval of convergence is $1 < x \leq 3$.

3:

First,

$f(x) = (1 + x)^{1/3}$,
$f'(x) = (1/3)(1 + x)^{-2/3}$,
$f''(x) = (1/3)(-2/3)(1 + x)^{-5/3}$, etc.

So, since we want a Taylor polynomial centered at $x = 0$, we try

$$f(0) + f'(0)x/1! + f''(0)x^2/2! + f'''(0)x^3/3! + \cdots$$

Taking just the first three nonzero terms, we get $1 + x/3 - (1/9)x^2$.

4:

(a)

$$f(x) = \frac{x^5}{9+x^2} = \frac{x^5}{9}\frac{1}{1-(-x^2/9)} = \frac{x^5}{9}\sum_{k=0}^{\infty}(-x^2/9)^k = \sum_{k=0}^{\infty}(-1)^k x^{2k+5}/9^{k+1}$$

for $|x^2/9| < 1$ or $|x| < 3$. If $|x| = 3$, $\sum_{k=0}^{\infty}(-x^2/9)^k$ diverges by the n-term test (the terms don't have limit 0), so the interval of convergence is $-3 < x < 3$.

5: Since $x(t) = 2-\cos(t)$ and $y(t) = -1+\sin(t)$, we have $(x-2)^2+(y+1)^2 = \cos^2(t)+\sin^2(t) = 1$, so we get $(x-2)^2 + (y+1)^2 = 1$; i.e., the equation of a circle of radius 1 with center at $(2, -1)$. The parameterization at $t = 0$ puts us at $(1, -1)$. As t starts to increases, x and y will increase, so we are going clockwise around the circle.

6:

(a) Look at the term $f^{(66)}(0)x^{66}/66!$ in the given series

$$f(x) = \sum_{k=0}^{\infty}\frac{(-1)^k}{4^k(k+1)}x^{2k}.$$

We see that it is

$$f^{(66)}(0)x^{66}/66! = \frac{(-1)^{33}}{4^{33}(33+1)}x^{2\cdot 33},$$

so

$$f^{(66)}(0) = 66!\frac{(-1)^{33}}{4^{33}34}.$$

(b)

$$f'(x) = \frac{d}{dx}\sum_{k=0}^{\infty}\frac{(-1)^k}{4^k(k+1)}x^{2k} = \sum_{k=1}^{\infty}\frac{(-1)^k 2k}{4^k(k+1)}x^{2k-1}$$

7:

(a) First, $dx/dt = e^{t-1} + te^{t-1}$, $dy/dt = 2t - 3$, so at $t = 1$ we have $x(1) = 1 + 1e^{1-1} = 2$, and $y(1) = 1^2 - 3 \cdot 1 = -2$, with $dy/dx = (dy/dt)/(dx/dt) = (2t-3)/(e^{t-1} + te^{t-1})$ which at $t = 1$ is $-1/2$, so the tangent line is $y - y(1) = (-1/2)(x - x(1))$ or $y = (-1/2)x - 1$.

(b)

$$\int_0^4 \sqrt{(dx/dt)^2 + (dy/dt)^2}\,dt = \int_0^4 \sqrt{(e^{t-1} + te^{t-1})^2 + (2t-3)^2}\,dt$$