M107 Project Instructions

Due Date: Thursday, April 15, 2004 (since extended to Tuesday, April 20)

Guidelines: This project is a group project which is based on infinite series, in chapter 8 of the book, but you will also need to other resources to carry out the project. Be sure to give a proper citation for any information that you include in your project write up that is not original to you. Also, your grade will be based partly on the quality of your written work. Use complete sentences, good grammar, correct spelling and correct punctuation. The paper you turn in should be a mix of equations, formulas and prose. Illustrations and tables should be clearly labeled. You should write your answers in such a way that it can be read and understood by anyone who knows the material for this course. This is your target audience, not the instructor. Finally, neatness counts, so the project should be neatly typed or written on good paper (not torn from a notebook). Your write up should note simply be numbered answers to the numbered items listed below; they are numbered just to make it easier for you to see what your project write up must cover. Instead, your write up should read as a coherent whole, an essay, into which the answers to the items listed are incorporated.

About Group Projects: To get everyone involved and the group functioning smoothly, it is a good idea to meet as early as possible to arrange meeting times, etc. It might be helpful to bear in mind that there are at least four roles to played by various participants at various times: the chair, reporter, scheduler and scribe. The role of the chair is to try to get everyone involved and make sure everyone is understanding the ideas developed by the group. The reporter jots down the ideas of the group as they are discussed. The scheduler finds times and places where everyone in the group can meet, and finally, the scribe writes up the final report for the group. These jobs can be rotated on a per meeting basis if the group wishes. However, everyone should proofread the final draft and help in the other duties as they see fit.

Participation: When the project is turned in, students may be asked to evaluate the level of participation by other group members by way of a project participation report to be filled out by each member individually and turned in to the recitation instructor.

Grading: Projects count 40 points. Points will be assigned as follows:

Completeness: 10
Correctness: 5
Grammar: 5
Full sentences: 5
Spelling: 5
Neatness: 5
Originality: 5

The Project: The number π turns up everywhere in science, technology and mathematics. This project concerns our developing ability to understand and compute π. Your write up should include the following items:

(1) A definition of π.
(2) Speculation on why we use the symbol π to denote this quantity.
(3) A discussion of at least two ancient approximations for π.
(4) An explanation of how the Taylor series for arctan can be used to give an alternating series converging to $\pi/4$, and hence an estimate for π. (This was known to Leibniz and possibly Gregory in the 1600s.) Using facts about alternating series, discuss the error you get by taking only the first n terms of this series for π. About how many terms are needed to get d digits of accuracy? Confirm your answer with a table showing the approximation you get, and the error, using various numbers of terms.
(5) An explanation of how the Taylor series for arcsin can be used to give an infinite series converging to $\pi/6$, and hence an estimate for π. (This was first done by Newton, in the 1600s, as an application after he worked out the Binomial Theorem for noninteger exponents.) Show that the error you get by taking only the first n terms of this series for π is less than 4^{-n}. [Hint: Bound the sum of the terms beyond the nth term by a geometric series.] Use the error estimate to determine how many terms would be sufficient to get d digits of accuracy. Run some numerical experiments to see how accurate an estimate for π the series actually gives, using the first 5, 10, 15, and 20 terms.
(6) A demonstration of using an improper integral to get an upper bound for the series $\Sigma_{k=n+1}^{\infty} k^{-2}$, as a function of n. Euler proved that $\sqrt{6\Sigma_{k=1}^{\infty} k^{-2}} = \pi$. Show that

$$\pi - \sqrt{6\Sigma_{k=1}^{n} k^{-2}} < (\pi^2/6 - \Sigma_{k=1}^{n} k^{-2})/(\pi/6) < 2(\Sigma_{k=n+1}^{\infty} k^{-2});$$

use this to estimate how many terms would suffice in Euler's series to estimate π to d digits of accuracy. Use a table to show what actually happens.
(7) A comparison of the methods of items 4, 5 and 6. Which in your estimation is most efficient? Justify your judgment.
(8) Comments on recent work.