M104, Practice Exam 3

Exam 3 is on Wednesday, March 27, 2013 in class in our usual classroom. The problems on the exam will be modeled on the ones here. Please work out the problems here in preparation for a class discussion on Monday, March 25. For each problem, you must show enough work to justify your answer.

(1) (35 points) In each case, either answer the question and justify your answer, or explain why there is not enough information to obtain an answer. In particular, when there is enough information to obtain an answer, your justification should explicitly indicate if you are applying the EVT (Extreme Value Theorem, p. 305), or the CPT (Critical Point Theorem, p. 307) and either the 1DT (1st derivative test, p. 265) or the 2DT (2nd derivative test, p. 281).

(a) Find all absolute extrema for a function $y = f(x)$ on the interval $10 \leq x \leq 100$, given that the critical numbers for $f(x)$ are $x = 20$ and $x = 30$, and given that $f(10) = 150$, $f(20) = 75$, $f(30) = -15$ and $f(100) = 35$.

(b) Find all absolute extrema for a function $y = f(x)$ which is continuous on the interval $10 \leq x \leq 100$, given that the critical numbers for $f(x)$ are $x = 20$ and $x = 30$, and given that $f(10) = 150$, $f(20) = 75$, $f(30) = -15$ and $f(100) = 35$.

(c) Find all absolute extrema for a function $y = f(x)$ continuous on the interval $0 < x < 6$, given that the critical numbers for $f(x)$ on the interval are $x = 2$ and $x = 4$, and given that $f(2) = 2$ and $f(4) = 4$.

(d) Find all absolute extrema for a function $y = f(x)$ continuous on the interval $3 < x < 5$, given that the only critical number for $f(x)$ on the interval is $x = 4$ and that $f(4) = 8$.

(e) Determine whether or not $y = f(x)$ has any absolute extrema on the interval $1 < x < 3$ and if so, for each one determine whether it is an absolute maximum or an absolute minimum given that $f(x)$ is a polynomial, the only critical number for $f(x)$ on the interval is $x = 2$ and $f''(2) < 0$.

(f) Determine whether or not $y = f(x)$ has any absolute extrema on the interval $0 < x < 4$ and if so, for each one determine whether it is an absolute maximum or an absolute minimum given that $f(x)$ continuous on the interval, the only critical number for $f(x)$ on the interval is $x = 2$ and $f''(2) = 0$.

(g) Find all absolute extrema for $y = f(x)$ on the interval $0 < x < 4$ and determine what kind of extremum each one is, given that $y = f(x)$ is a polynomial, the only critical number for $f(x)$ on the interval is $x = 2$ and given that $f'(1) < 0$ and $f'(3) > 0$.

(2) (20 points) Consider the function $y = f(x)$ graphed at right.
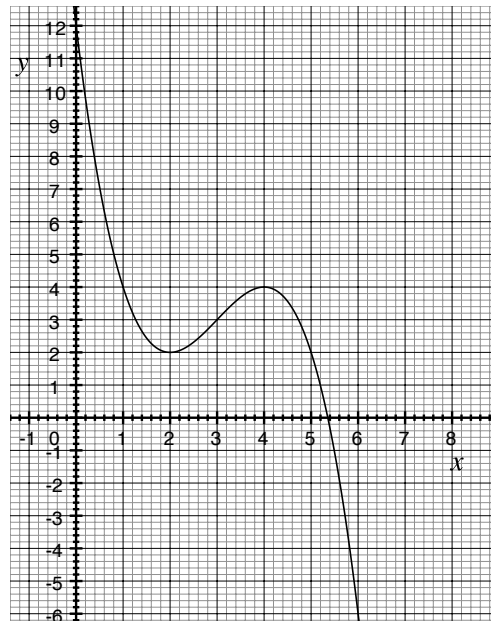
(a) Find the open interval of x-values for which $f(x)$ is concave down.

(b) Find the open interval of x-values for which $f''(x) < 0$.

(c) Find the coordinates of all points of inflection.

(d) If for $x > 2$ the function $f(x)$ gives the profit of a company at a production level x, find the point of diminishing marginal returns.

(3) (8 points) A local club is arranging a charter flight to Rome at \$1600 each, except each passenger gets a refund of \$10 for each passenger above 90 (so everybody gets \$10 off if there are 91 passengers and \$20 off if there are 92 passengers, etc.). Find the number of passengers that maximizes the total revenue. Justify your answer.

(4) (7 points) A paint company has steady sales of 1800000 gallons of paint per year. It costs the paint company \$1000 in set up costs each time it makes a batch of paint. In addition to set up costs, it costs the company \$1 per gallon per year to store a gallon of paint. How many production runs per year should the company have in order to meet its demand while minimizing its costs? Use calculus to justify your answer.

(5) (6 points) Assume in a certain business situation that elasticity of relative demand is $E = 0.5$.

(a) Will revenue increase or decrease if unit price is increased?

(b) If price is increased by 2%, approximately how much (in percentage terms) will demand decrease?

(6) (8 points) Use implicit differentiation to find the slope of the tangent line through the point $x = 3$ and $y = 2$ if $y^3 + xy + x^2 = 23$. Show your work.

(7) (8 points) Find the marginal revenue $\frac{dR}{dt}$ if $\frac{dp}{dt} = 12$ and $\frac{dq}{dt} = -5$ when $p = 10$ and $q = 15$. Show your work.

(8) (8 points) Let $f(x) = 2x^6 - 24x^5 + 75x^4$, so $f'(x) = 12x^5 - 120x^4 + 300x^3 = 12x^3(x-5)^2$ and $f''(x) = 60x^4 - 480x^3 + 900x^2 = 60x^2(x-3)(x-5)$. Show how you obtain your answer.

(a) Find all open intervals on which $f(x)$ is concave up with $f''(x) \neq 0$.

(b) Find all open intervals on which $f(x)$ is concave down with $f''(x) \neq 0$.

(c) Find all x-values at which $f(x)$ has a point of inflection.