(1) (This problem is like Example 6.8 on p. 379.) Use the substitution $u = 4-2x$ to convert the given integral into an integral in terms of u only: $\int_0^1 2x\sqrt{4-2x}\,dx$. Solution: $du = -2dx$ and $2x = 4-u$, hence $\int_0^1 2x\sqrt{4-2x}\,dx = -\int_4^2 (4-u)\sqrt{u}\,du/2 = (1/2)\int_2^4 (4-u)\sqrt{u}\,du$.

(2) (This problem is like homework problem #29, 5.1.) Let R be the region bounded by the curves $y = \sqrt{x}$, $y = 2-x$, and $y = 0$. Solution: The area is either $\int_0^1 ((2-y)-y^2)\,dy$ or $\int_0^1 \sqrt{x}\,dx + \int_1^2 (2-x)\,dx$.

(3) Let R be the region bounded by the curves $y = 1$, $x = 1$, and $y = 100/x^2$. Solution: the volume of the solid formed by revolving R about the y-axis, is $\int_1^{100} (\pi 100/y - \pi 1^2)\,dy$ using horizontal slices, and using shells it is $\int_1^{10} (100/x^2 - 1)2\pi x\,dx$.

(4) (This is homework problem #6, 5.6.) A force of 10 pounds stretches a spring 2 inches. Set up an integral for the work done in stretching the spring 3 inches beyond its natural length. Solution: $F = kx$, so $10 = k/6$ (since 2 inches is 1/6 feet), hence $k = 60$. Now the work, in foot-pounds, is $\int_0^{1/4} 60x\,dx$.

(5) (This problem is like homework problem #31, 6.4.) The initial temperature of the porridge is 150°F. The ambient temperature is 50°F. After 10 minutes, the porridge is 120°F. How long does it take to cool from 150°F to 99°F? Solution: the temperature difference decays exponentially, so, if $y(t)$ is the temperature of the porridge as a function of time in minutes t after 9:00 (so $t = 0$ indicates 9:00), then $y(t) - 50 = (y(0) - 50)e^{kt}$. Since $y(0) = 150$, this is $y(t) - 50 = 100e^{kt}$ or $y(t) = 50 + 100e^{kt}$. We need k, but $120 = y(10) = 50 + 100e^{10k}$, so $k = (\ln 0.7)/10$. Now solve $99 = 50 + 100e^{kt}$ for t to get $\ln 0.49 = (\ln 0.7)t/10$ and thus $t = 20$. I.e., the porridge is just right at 9:20.

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