TA: ____________________

Instructor: ____________________ NAME: ____________________

*Instructions*: Show all of your work and clearly explain your answers. This is particularly important on problems with a numerical answer, to allow the possibility of partial credit. No books or written notes are allowed during the exam, but you may use your calculator. Also note that this exam should have 6 pages; please check that it does.

| Problem | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Totals |
|---|---|---|---|---|---|---|---|---|---|---|
| Points | 24 | 20 | 24 | 20 | 22 | 22 | 20 | 24 | 24 | 200 |
| Score | | | | | | | | | | |

[1] (24 points) Let $R$ be the region enclosed by $y = x^2$ and $y = 2x + 3$.

(a) (12 points) Sketch $R$ and set up an integral for the area of $R$. You do not need to evaluate the integral.

(b) (12 points) Set up an integral for the volume obtained by revolving $R$ about the line $x = -2$. You do not need to evaluate the integral.

[2] (20 points) Express as a definite integral the work required to pump all of the oil (having density equal to 40 pounds per cubic foot) out the top of a completely full spherical tank of radius 10 feet. You do not have to evaluate this integral.

[3] (24 points) Evaluate the following two indefinite integrals. Show all of your steps.

(a) (12 points) $\displaystyle\int \frac{1}{x^2 + 4x + 13} dx$

(b) (12 points) $\displaystyle\int x^2 \ln(x)\, dx$

[4] (20 points) Do three things: Explain why the function $f(x) = x^3 + x - 3$ is invertible, show that the point $(7, 2)$ is on the graph of $y = f^{-1}(x)$, and find the equation of the line tangent to the graph of $y = f^{-1}(x)$ at the point $(7, 2)$.

[5] (22 points) Determine whether there is a finite area under the graph of the curve $y = (x^2 + 1)^{-1}$ over the interval $0 \leq x < \infty$. If so, determine the area exactly. Explain your answer.

[6] (22 points) Determine the sum of the given power series at $x = 1/3$. Explain how you obtain your answer. [Hint: find a function $f(x)$ whose Taylor series is the given series; the sum is then $f(1/3)$.]

$$\sum_{k=1}^{\infty} kx^{k-1} = 1 + 2x + 3x^2 + 4x^3 + \cdots$$

[7] (20 points) This problem concerns parametric equations. The two parts are independent.

(a) (8 points) Give parametric equations $x = f(t)$, $y = g(t)$ for the line segment from the point $(2, 3)$ to $(-7, 5)$ such that $(f(0), g(0)) = (2, 3)$, and $(f(1), g(1)) = (-7, 5)$.

(b) (12 points) Determine the exact distance a particle moves from time $t = 0$ to time $t = 10$, where its position as a function of time $t$ is given by $x = \sin(t^2)$ and $y = \cos(t^2)$.

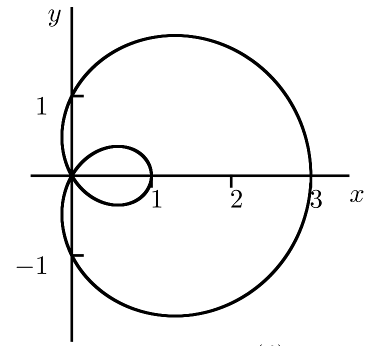
[8] (24 points) Let $f(x) = \sum_{k=1}^{\infty} (-1)^k \frac{x^k}{k7^k}$.

(a) (12 points) Determine the interval of convergence of $\sum_{k=1}^{\infty} (-1)^k \frac{x^k}{k7^k}$. Pay particular attention to behavior at the endpoints of the interval.

(b) (12 points) Estimate the value of $f(2)$ to within 0.01. Justify why your answer has the required accuracy.

[9] (24 points) Consider the curve defined for $0 \le \theta \le 2\pi$ in polar coordinates by $r = 1 + 2\cos(\theta)$.

(a) (12 points) Find the equation of the line tangent to the curve at the point $\theta = \pi/2$.




$r = 1 + 2\cos(\theta)$

(b) (12 points) Set up an integral for the area enclosed by the inner loop of the curve. You do not need to evaluate the integral.