

TA: \_\_\_\_\_

NAME: \_\_\_\_\_

*Instructions:* Show all of your work and clearly explain your answers. This is particularly important on problems with a numerical answer, to allow the possibility of partial credit. No books or written notes are allowed during the exam, but you may use your calculator. Also note that this exam should have 3 pages; please check that it does.

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Problem	1	2	3	4	5	Totals
Points	20	20	20	20	20	100
Score						

[1] (20 points) (a) Give an example of a series which is only conditionally convergent. Justify your answer.

(b) Determine whether or not the series given below is absolutely convergent. Justify your answer.

$$\sum_{k=1}^{\infty} (-1)^{k+1} \frac{1}{k!}$$

[2] (20 points) Using facts about alternating series, determine an  $n \geq 0$  such that the difference between the sum  $S$  of the alternating series given below and its partial sum  $S_n = \sum_{k=1}^n (-1)^{k+1} \frac{1}{k!}$  is less than 0.01. Justify your answer.

$$\sum_{k=1}^{\infty} (-1)^{k+1} \frac{1}{k!}$$

[3] (20 points) (a) Write down the Taylor polynomial  $P_4(x)$  for  $f(x) = e^x$ .

(b) The series given below is obtained by evaluating the Taylor series of some function  $g(x)$  at a particular value of  $x$ . Find  $g(x)$  by modifying the Taylor series of  $e^x$ , and then find the sum  $S$  of the series exactly by evaluating  $g(x)$  at an appropriate value of  $x$ . Explain your answer. (Note that the series starts with  $k = 1$ .)

$$\sum_{k=1}^{\infty} \frac{2^{k+1}}{k!}$$

[4] (20 points) Determine the radius and interval of convergence of the series given below. Justify your answers, and make sure to explain what happens at both endpoints of the interval.

$$\sum_{k=1}^{\infty} \frac{(x-2)^k}{k3^k}$$

[5] (20 points) Let  $C$  be the curve given parametrically by  $x(t) = t^2$  and  $y(t) = t^3 - t$ .

(a) Determine the  $x - y$  equation of the tangent line to the curve at the point  $x = 4$ ,  $y = 6$ .

(b) Find all values of  $t$  such that the curve has a horizontal tangent line.