MATH 428/828 (circle one) **Exam 1** **Spring Semester, 2013**

Name:____________________ Any 4 digit PIN:__________ Score:________

1(25pts) Consider the linear programming problem

Minimize $z = x_1 + 2x_2$
subject to $x_1 + x_2 \geq 2$
$x_1 + 3x_2 \geq 4$
$x_1 \geq 0,\ x_2 \geq 0.$

(a) Sketch the feasible region and use the graphical method to solve the problem.

(b) Formulate the problem as an augmented (equality) linear programming problem.

(c) Use the Big-M method to write down *only* the initial tableau for the simplex method, and explain why having surplus variables is not sufficient in general and why you need to introduce artificial variables.

(d) **For M828 Student Only:** Find *only* the initial feasible echelon form of the augmented Big-M problem and its corresponding feasible solution and value. **Do NOT find any subsequent echelon form or solution.**

2(25pts) The augmented matrix form for a LP problem: max. $z = c^T x$ subject to $Ax \leq b,\ x \geq 0$ is given as follows:

		z	x_1	x_2	x_3	x_4	rhs
	z	1	-4	-3	0	0	0
basic variable	x_3	0	2	1	1	0	10
	x_4	0	2	3	0	1	18

(a) Determine the vectors c, b and the matrix A.

(b) Find the optimal solution of the problem by using the tableau simplex method only.

(c) **For M828 Student Only:** Write the dual LP problem and its solution obtained from (b) as the shadow price of the primal LP problem.

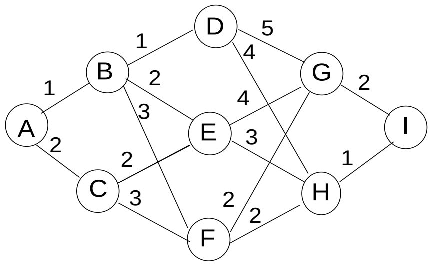
3(25pts) Consider a staged bipartite graph as shown with the assumption that each edge weight represents some type of cost.

(a) Use the *graphical* dynamical programming method to find the optimal path or paths from state **A** to state **I**.

(b) The backward iterative method to find the optimal path or paths from state **A** to state **I** looks like below for the first two iterations. Complete the remaining iterations and find the optimal solution.

S_4	$f_4^*(S_4)$	S_5^*
G	2	I
H	1	I

	$f_3(S_3, S_4) = C_{S_3S_4} + f_4^*(S_4)$			
$S_3 \backslash S_4$	G	H	$f_3^*(S_3)$	S_4^*
D	7	5	5	H
E	6	4	4	H
F	4	3	3	H



4(25pts) Consider a game having the following payoff table for Player 1:

		Player 2	
	Strategy	**1**	**2**
	1	-1	5
Player 1	**2**	7	1
	3	9	-3

(a) Let $G(y_1) = \max_{\{x_i \geq 0, \sum x_i = 1\}} E(x, y)$ with E being the expected pay-off to Player 1, $x = (x_1, x_2, x_3)$ and $y = (y_1, y_2)$ the mixed strategies for Player 1 and Player 2 respectively. Sketch the graph of $E = G(y_1)$.

(b) Use the graphical method to find the game value $v = \min_{\{y_j \geq 0, \sum y_j = 1\}} \max_{\{x_i \geq 0, \sum x_i = 1\}} E(x, y)$ and the optimal strategy y^* for Player 2.

(c) Find the optimal strategy x^* for Player 1.

(d) Set up one linear programming problem for the game problem, and indicate how the optimal solution can be found from solving your LP problem. **Do not solve the linear programming problem.**

End