

of this age group for the future. AmeriBank wants to interview 2,000 customers with at least 20 percent from the first age group, at least 27.5 percent from the second age group, at least 15 percent from the third age group, and at least 15 percent from the fourth age group.

Rob wants to ensure that the survey includes a mix of customers who know the Internet well and those that have less exposure to the Internet. To ensure that AmeriBank obtains the correct mix, he wants to interview at least 15 percent of customers

from the Silicon Valley where Internet use is high, at least 35 percent of customers from big cities where Internet use is medium, and at least 20 percent of customers from small towns where Internet use is low.

Sophisticated Surveys has performed an initial analysis of these survey requirements to determine the cost of surveying different populations. The costs per person surveyed are listed in the following table:

| Region         | Age Group |          |          |             |
|----------------|-----------|----------|----------|-------------|
|                | 18 to 25  | 26 to 40 | 41 to 50 | 51 and over |
| Silicon Valley | \$4.75    | \$6.50   | \$6.50   | \$5.00      |
| Big cities     | \$5.25    | \$5.75   | \$6.25   | \$6.25      |
| Small towns    | \$6.50    | \$7.50   | \$7.50   | \$7.25      |

Sophisticated Surveys explores the following options cumulatively.

- Formulate a linear programming model to minimize costs while meeting all survey constraints imposed by AmeriBank.
- If the profit margin for Sophisticated Surveys is 15 percent of cost, what bid will they submit?
- After submitting its bid, Sophisticated Surveys is informed that it has the lowest cost but that AmeriBank does not like the solution. Specifically, Rob feels that the selected survey population is not representative enough of the banking customer population. Rob wants at least 50 people of each age group surveyed in each region. What is the new bid made by Sophisticated Surveys?
- Rob feels that Sophisticated Survey oversampled the 18- to 25-year-old population and the Silicon Valley population. He imposes a new constraint that no more than 600 individuals can be surveyed from the 18- to 25-year-old population and no more than 650 individuals can be surveyed from the Silicon Valley population. What is the new bid?
- When Sophisticated Surveys calculated the cost of reaching and surveying particular individuals, the company thought that reaching individuals in young populations would be easiest. In a recently completed survey, however, Sophisticated Surveys learned that this assumption was wrong. The new costs for surveying the 18- to 25-year-old population are listed below.

#### Region survey cost per person

|                |        |
|----------------|--------|
| Silicon Valley | \$6.50 |
| Big cities     | \$6.75 |
| Small towns    | \$7.00 |

Given the new costs, what is the new bid?

- To ensure the desired sampling of individuals, Rob imposes even stricter requirements. He fixes the exact percentage of people that should be surveyed from each population. The requirements are listed below:

#### Population percentage of people surveyed

|                |     |
|----------------|-----|
| 18 to 25       | 25% |
| 26 to 40       | 35% |
| 41 to 50       | 20% |
| 51 and over    | 20% |
| Silicon Valley | 20% |
| Big cities     | 50% |
| Small towns    | 30% |

By how much would these new requirements increase the cost of surveying for Sophisticated Surveys? Given the 15 percent profit margin, what would Sophisticated Surveys bid?

## CASE 4.3 ASSIGNING STUDENTS TO SCHOOLS

The Springfield school board has made the decision to close one of its middle schools (sixth, seventh, and eighth grades) at the end of this school year and reassign all of next year's middle school students to the three remaining middle

schools. The school district provides bussing for all middle school students who must travel more than approximately a mile, so the school board wants a plan for reassigning the students that will minimize the total bussing cost. The

### CASE 4.3 ASSIGNING STUDENTS TO SCHOOLS

9

annual cost per student of bussing from each of the six residential areas of the city to each of the schools is shown in the following table (along with other basic data for next

year), where 0 indicates that bussing is not needed and a dash indicates an infeasible assignment.

| Area             | No. of Students | Percentage in 6th Grade | Percentage in 7th Grade | Percentage in 8th Grade | Bussing Cost per Student |          |          |
|------------------|-----------------|-------------------------|-------------------------|-------------------------|--------------------------|----------|----------|
|                  |                 |                         |                         |                         | School 1                 | School 2 | School 3 |
| 1                | 450             | 32                      | 38                      | 30                      | \$300                    | 0        | \$700    |
| 2                | 600             | 37                      | 28                      | 35                      | —                        | \$400    | \$500    |
| 3                | 550             | 30                      | 32                      | 38                      | \$600                    | \$300    | \$200    |
| 4                | 350             | 28                      | 40                      | 32                      | \$200                    | \$500    | —        |
| 5                | 500             | 39                      | 34                      | 27                      | 0                        | —        | \$400    |
| 6                | 450             | 34                      | 28                      | 38                      | \$500                    | \$300    | 0        |
| School capacity: |                 |                         |                         |                         | 900                      | 1,100    | 1,000    |

The school board also has imposed the restriction that each grade must constitute between 30 and 36 percent of each school's population. The above table shows the percentage of each area's middle school population for next year that falls into each of the three grades. The school attendance zone boundaries can be drawn so as to split any given area among more than one school, but assume that the percentages shown in the table will continue to hold for any partial assignment of an area to a school.

You have been hired as an operations research consultant to assist the school board in determining how many students in each area should be assigned to each school.

- Formulate a linear programming model for this problem.
- Solve the model.
- What is your resulting recommendation to the school board?

After seeing your recommendation, the school board expresses concern about all the splitting of residential areas among multiple schools. They indicate that they "would like to keep each neighborhood together."

- Adjust your recommendation as well as you can to enable each area to be assigned to just one school. (Adding this restriction may force you to fudge on some other constraints.) How much does this increase the total bussing cost? (This line of analysis will be pursued more rigorously in Case 11.4.)

The school board is considering eliminating some bussing to reduce costs. Option 1 is to eliminate bussing only

for students traveling 1 to 1.5 miles, where the cost per student is given in the table as \$200. Option 2 is to also eliminate bussing for students traveling 1.5 to 2 miles, where the estimated cost per student is \$300.

- Revise the model from part (a) to fit Option 1, and solve. Compare these results with those from part (c), including the reduction in total bussing cost.
- Repeat part (e) for Option 2.

The school board now needs to choose among the three alternative bussing plans (the current one or Option 1 or Option 2). One important factor is bussing costs. However, the school board also wants to place equal weight on a second factor: the inconvenience and safety problems caused by forcing students to travel by foot or bicycle a substantial distance (more than a mile, and especially more than 1.5 miles). Therefore, they want to choose a plan that provides the best trade-off between these two factors.

- Use your results from parts (c), (e), and (f) to summarize the key information related to these two factors that the school board needs to make this decision.
- Which decision do you think should be made? Why?

*Note:* A data file for this case is included on the CD-ROM for your convenience. Also note that this case will be continued in later chapters (Cases 6.3 and 11.4), so we suggest that you save your analysis, including your basic model.