**MATH 428 Final Exam** **Spring Semester, 2010**

Name:_____ Score:_____

**Closed book exam, but calculators are allowed.**

1(40pts) The coach of an age group swim team needs to assign swimmers to a 200-yard medley relay team to send to the Junior Olympics. Since most of his best swimmers are very fast in more than one stroke, it is not clear which swimmer should be assigned to each of the four strokes. The five fastest swimmers and the best times (in seconds) they have achieved in each of the stroke categories (for 50 yards) are

| **Stroke** | **Alex** | **Bobby** | **Chris** | **David** | **Ed** |
|---|---|---|---|---|---|
| Backstroke | 37.7 | 32.9 | 33.8 | 37.0 | 35.4 |
| Breaststroke | 43.4 | 33.1 | 42.2 | 34.7 | 41.8 |
| Butterfly | 33.3 | 28.5 | 38.9 | 30.4 | 33.6 |
| Freestyle | 29.2 | 26.4 | 29.6 | 28.5 | 31.1 |

Formulate a BIP model to minimize possible time time for the 200-yard medley relay. (*Hint:* Let $x_{ij} = 1$ for swimmer $j$ to swim stroke $i$ and $x_{ij} = 0$ otherwise. **Do not solve for the optimal solution.**)

2(40pts) A jewelry store always stocks up it 1-carat diamonds at the end of each month so that it has exactly two 1-carat diamonds for the beginning of each month. That is, if the number of stone at the end of a month is $X_n = 0, 1, 2$, the store will order $2, 1, 0$ stones for the coming month. It is estimated that the monthly demand for its 1-carat diamonds are independent and distributed according to

$$P\{D = 0\} = \frac{1}{4}, \quad P\{D = 1\} = \frac{1}{4}, \quad P\{D = 2\} = \frac{1}{4}, \quad P\{D \geq 3\} = \frac{1}{4}.$$

(a) Construct the (one-step) transition matrix.

(b) Find the steady-state probabilities of the state of this Markov chain.

(c) Find the expected first passage time from the store not having to order any to having to order 2 diamonds.

3(40pts) A maintenance person has the job of keeping 3 machines in working order. The amount of time that a machines works before breaking down has an exponential distribution with a mean of 2 weeks. The time then spent by the maintenance person to repair the machine has an exponential distribution with a mean of 2 days.

(a) Show that this process fits the birth-and-death process by defining the states, specifying the values of the $\lambda_n$ and $\mu_n$, and the displaying the rate diagram.

(b) Find probabilities $p_n$ for $n \geq 0$.

(c) Determine the proportion of time that the maintenance person is busy.

(d) Determine the expected time that a machine will be out of service.

4(40pts) You want to choose between decision alternative $A_1$ and $A_2$ in the decision tree on the right, but you are uncertain about the value of the probability $p$.

(a) For $p = 0.25$, what is the optimal alternative?

(b) Determine the range of $p$ so that $A_1$ is always the optimal alternative.

5(40pts) Consider a game having the following payoff table for Player 1:

| | | **Player 2** | |
|---|---|---|---|
| **Strategy** | | **1** | **2** |
| **Player 1** | **1** | 2 | 1 |
| | **2** | 1 | 3 |

(a) Set up a linear programming problem for Player 2's optimal strategy.

(b) Use the Big M method to work through the simplex method step by step to find the optimal strategies for both players.

**End**