MATH 428 Final Exam **Spring Semester, 2009**

Name:_____ Score:_____

Closed book exam, but calculators are allowed.

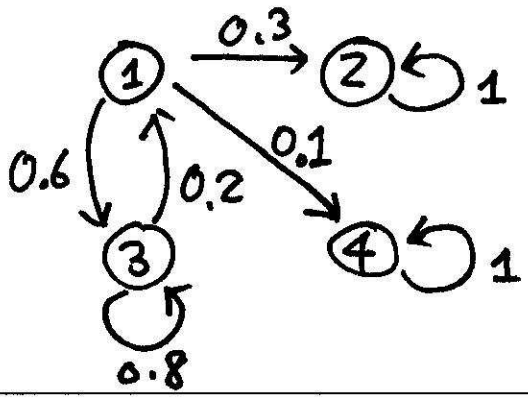
1(40pts) A young couple, Eve and Steve, want to divide their household chores between them so that each has at least two but no more than three tasks, and they agree on that the total time they spend on the house chores is kept to a minimum. In addition, no one wants to end up doing both cooking and dishwashing. Their efficiencies on the tasks differ, where the time in hours each would need to perform the task is given in the table below:

	Time Needed Per Week				
	Bathroom Cleaning	**Vacuuming**	**Laundry**	**Cooking**	**Dishwashing**
Eve	1.5	4	2	8	4
Steve	1	5	2.5	7	3.5

(a) Formulate a BIP model for this problem. (*Hint:* Let $x_i = 1$ for Eve to do task i and $x_i = 0$ for Steve to do it.)

(b) You are given the partial information that the optimal solution tells them that Steve need do bathroom cleaning, Eve need do vacuuming and laundry. Complete the solution using the Branch-Bound method.

2(30pts) A Markov process is given by the transition diagram shown. Find the probability that State 3 would be absorbed by State 4.



3(30pts) In a small area of the Arizona desert, there are two desert shrubs, plant A and plant B. It was observed that in any given year the area is either free of the plants (open state) or dominated by one of the plants, and the yearly transition probability matrix is given on the right.

(a) Find the steady-state probability that the area would be open.

(b) Find the expected time (in years) that Plant B would take to dominate the area from either being Plant A dominated or open.

	O	A	B
O	0.6	0.4	0
A	0.05	0.9	0.05
B	0.15	0	0.85

4(25pts) Consider the birth-death process with $\mu_n = 2$ for all $n \geq 1$ and $\lambda_0 = 3$, $\lambda_1 = 2$, $\lambda_2 = 1$, and $\lambda_n = 0$ for $n \geq 3$.

(a) Display the rate diagram.

(b) Find probabilities p_n for $n \geq 0$.

(c) Find L, L_q, W, and W_q.

(*Continue on Back ...*)

5(25pts) Consider an assembly line operation consisting of 3 sequential processes, each must work for the assembly line to function. The reliability of the line can be improved by staffing more independent quality controllers than the necessary minimum of one controller at each of the processes. The following table gives the probability that the respective process will function if it has 1, 2, or 3 controllers.

	Probability of Functioning		
Controllers	**Process 1**	**Process 2**	**Process 3**
1	0.5	0.6	0.7
2	0.6	0.7	0.8
3	0.8	0.8	0.9

The cost (in thousands of dollars) of having 1, 2, or 3 controllers in the respective processes is given by the following table:

	Cost		
Controllers	**Process 1**	**Process 2**	**Process 3**
1	1	2	1
2	2	4	3
3	3	5	4

Because of budget limitation, no more than \$6,000 can be expended. Use dynamic programming to determining how many quality controllers should be stationed in each process to maximize the reliability of the assembly line.

6(25pts) Consider a game having the following payoff table for player 1:

		Player 2		
Strategy		**1**	**2**	**3**
Player 1	**1**	3	-2	2
	2	-1	2	2

Use the graphical method to determined the game value and the optimal mixed strategy for each player according to the minimax criterion.

7(25pts) Use the Big M method to work through the simplex method step by step to solve the following problem.

Minimize $Z = 3x_1 + 2x_2 + 4x_3$

subject to

$$2x_1 + x_2 + 3x_3 \leq 60$$
$$3x_1 + 3x_2 + 5x_3 \geq 120$$
$$x_i \geq 0, \ i = 1, 2, 3.$$

End