

Name: \_\_\_\_\_

Score: \_\_\_\_\_

- 1(30pts) The Primo Insurance Company is introducing two new product lines: special risk insurance and mortgages. The expected profit is \$5 per unit on special risk insurance and \$2 per unit on mortgages. The management wishes to establish sales quotas for the new product lines to maximize total expected profit. The work requirements are as follows:

Department	Work-Hours per Unit		Work-Hours Available
	Special Risk	Mortgage	
Underwriting	3	2	2400
Administration	0	1	800
Claims	2	0	1200

- Determine the decision variables and formulate a linear programming model for this problem.
- Use the graphical method to find the optimal solution.
- Determine the shadow price with respect to underwriting.
- Identify unbinding constraint(s).
- Let  $x_3, x_4, x_5$  be the slack variables. What do  $x_i = 0, i = 3, 4, 5$  mean graphically? (It suffices to label these equations on your graph above.)
- Formulate the 0 iteration of the tabular form of the simplex method and complete the first iteration of the tabular form. Identify the CPF solution.
- Use the information from the graphical method above to determining the basic variables and the nonbasic variables at the (would be) final iteration of the tabular form of the simplex method.
- (Bonus 5 pts) If 100 work-hours were shifted from the Department of Administration to the Department of Underwriting, what would be the increase in profit?

- 2(10pts) Consider the game having the following payoff table.

		Player 2			
		1	2	3	4
Player 1	1	0	-3	-2	-4
	2	-4	-2	-1	1
	3	1	-1	2	0

- Use the minimax criterion to find the best strategy for each player. Is it a stable game?
- Identify and eliminate dominated strategies as far as possible. Make a list of the dominated strategies, showing the order in which you were able to eliminate them. Then show the resulting reduced payoff table with remaining strategies.

- 3(30pts) Consider the game having the following payoff table.

		Player 2		
		1	2	3
Player 1	1	5	0	2
	2	-1	4	3

- For each of the pure strategies available to player 2, list the expected payoff for player 1.
- Determine the expected payoff function for player 1 given a mixed strategy for the player 2.
- Formulate the problem of finding optimal mixed strategies according to the minimax criterion as a linear programming problem. (Do not solve it by graphical or simplex method.)
- Use the graphical procedure to determine the value of the game and the optimal mixed strategy for each player according to the minimax criterion.

(Continue on Next Page ...)

- 4(30pts) Management of the Telemore Company is considering developing and marketing a new product. It is estimated to be twice as likely that the product would prove to be successful as unsuccessful. If it were successful, the expected profit would be \$1,500,000. If unsuccessful, the expected loss would be \$1,800,000. A marketing survey can be conducted at a cost of \$300,000 to predict whether the product would be successful. Past experience with such surveys indicates that successful products have been predicted to be successful 80 percent of the time, whereas unsuccessful products have been predicted to be unsuccessful 70 percent of the time.
- (a) Formulate this problem by identifying the alternative actions, the states of nature, the payoff table when the market survey is not conducted, and the prior probabilities.
  - (b) Assuming the market survey is not conducted, use Bayes's decision rule to determine which decision alternative should be chosen.
  - (c) Use a probability tree diagram to find all posterior probabilities.
  - (d) Draw and properly label the decision tree. Include all the payoffs and the probabilities. (If you cannot find the required probabilities from part (c) above, give a hypothetical set of the probabilities and proceed to complete part (e) below.)
  - (e) Apply the backward induction procedure to find the optimal policy. What is the expected payoff of conducting a market survey excluding the cost?

---

**End**

## Math 428 Exam I Solution Key Spring 2002

1 (30pts)

Dept.	S. risk	Mort	Wkt. available
Underwriting	3	2	2400
Admin	0	1	800
claims	2	0	1200

(a)  $x_1$  — quota for s. risk.  $x_2$  — quota for mortgageMaximize  $Z = 5x_1 + 2x_2$ 

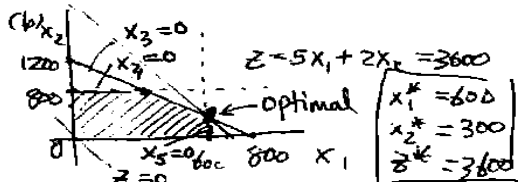
subject to

$$3x_1 + 2x_2 \leq 2400$$

$$2x_1 \leq 800$$

$$x_2 \leq 1200$$

$$x_1 \geq 0, x_2 \geq 0$$

(c) Underwriting  $3x_1 + 2x_2 = 2400 + \Delta b$   
Admin:  $2x_1 = 1200$ 

$$\Rightarrow x_1 = 600, x_2 = 300 + \frac{\Delta b}{2}$$

$$Z = 5x_1 + 2x_2 = 3600 + \Delta b \Rightarrow \frac{\Delta Z}{\Delta b} = \frac{\Delta b}{\Delta b} = 1$$

(d) Administration  $x_2 \leq 800$ (e) Let  $x_3, x_4, x_5$  be slack variables for constraints ①-③

iteration	basic variable	Eq		coeff.	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	right side	Ratio
0	$Z$	0	1	-5	-2	0	0	0	0	0	
	$x_3$	1	0	3	2	1	0	0	0	2400	800
	$x_4$	2	0	0	1	0	1	0	0	800	$\infty$
	$x_5$	3	0	2	0	0	0	1	1	1200	$600 < \min$
1	$Z$	0	1	0	-2	0	0	$\frac{5}{2}$	$-\frac{3}{2}$	3000	
	$x_3$	1	0	0	$\frac{2}{3}$	1	0	$-\frac{1}{3}$	$-\frac{1}{3}$	600	
	$x_4$	2	0	0	1	0	1	0	0	800	
	$x_1$	3	0	1	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	600	

BF/CPF:  $(x_1, x_2, x_3, x_4, x_5) = (600, 0, 600, 800, 0)$ (g) Basic variables:  $x_1, x_2, x_4$ . Nonbasic variables:  $x_3, x_5$ (h) Else (c),  $\Delta b = 100$ ,  $x_1^* = 600$ ,  $x_2^* = 300 + \frac{100}{2} = 350 < 800 - \Delta b = 700$ . Optimal solution stays at the same CPF with  $\Delta Z = \Delta b = 100$ .

2 (9pts)

	1	2	3	4	min
1	0	3	-2	-4	-4
2	-4	2	-1	-1	-4
3	1	-1	2	0	-1
max	1	-1	2	0	

$\uparrow$  min max.

 $\Rightarrow$  stable game.(b)  $P_2$ : ② dominates ③ $P_1$ : ③ " ① $P_2$ : ② " ④ $P_1$ : ③ " ② $P_2$ : ③ " ① $\Rightarrow$   $P_2$  w/ ②  
 $P_1$  w/ ③

3 (3pts)

		$x_1$	$y_2$	$y_3$
$p_1$	$x_1$	5	0	2
	$x_2$	-1	4	3

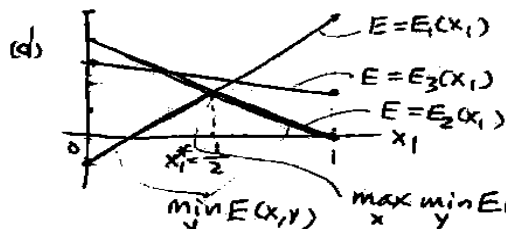
(a)  $N, v_2, v_{2H}$  Expected payoff

$$\begin{array}{l|l} (1, 0, 0) & 5x_1 - (1-x_2) = 6x_1 - 1 = E_1(x_1) \\ (0, 1, 0) & 4(1-x_1) = 4 - 4x_1 = E_2(x_1) \\ (0, 0, 1) & 2x_1 + 3(1-x_1) = 3 - x_1 = E_3(x_1) \end{array}$$

$$b) E(x, y) = y_1 E_1(x_1) + y_2 E_2(x_1) + y_3 E_3(x_1) = y_1(6x_1 - 1) + y_2(4 - 9x_1) + y_3(3 - x_1)$$

(c) maximize  $Z = x_3$  subject to:  $5x_1 - x_2 - x_3 \geq 0$

$$\begin{aligned} &+x_2 - x_3 \geq 0 \\ &2x_1 + 3x_2 - x_3 \geq 0 \\ &x_1 + x_2 = 1, \quad x_1 \geq 0, x_2 \geq 0 \end{aligned}$$



$$\begin{cases} E = E_1(x_1) = 6x_1 - 1 & \rightarrow 6x_1 - 1 = 4 - 4x_1 \\ E = E_2(x_1) = 4 - 4x_1 & \rightarrow 10x_1 = 5, \quad x_1^* = \frac{1}{2} \rightarrow x_2^* = \frac{1}{2} \end{cases}$$

$\Rightarrow$  Let  $(y_1^*, y_2^*, y_3^*)$  be the optimal dual solution.  $v = E(x_1^*) = z$   
 $\Rightarrow y_1^*(6x_1 - 1) + y_2^*(4 - 4x_1) + y_3^*(3 - x_1) \leq v = z$  for all  $x_1 \leq 1$   
 at  $x_1 = x_1^*$ ,  $E = v \Rightarrow y_3^* = 0$

and  $\Rightarrow y_1^*(6x_1, -1) + y_2^*(4 - 4x_1) = 2$  for all  $x_1 \in I$   
 $\Rightarrow (x_1^*, y_2^*) = (\frac{1}{2}, \frac{1}{2})$  for all  $x_1 \in I$

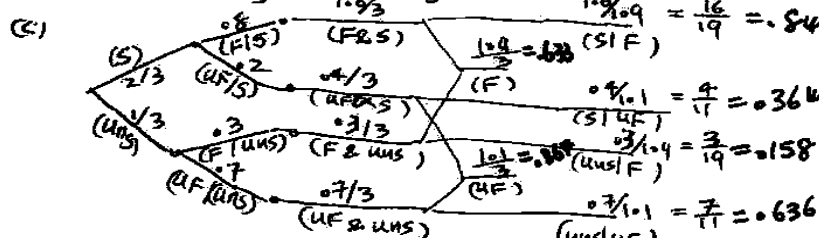
4 (30pts). (a)

(a) action	states of nature successful	unsuccessful
D2.M	1.5	-1.8
no D2.M	0	0
Prior Prob.	2/3	1/3

$$(b) E(DRM) = \frac{2}{3}(1.5) + \frac{1}{3}(1.8) = 1 + (-.6) = .4$$

$$E(\text{No DEM}) = 0$$

⇒ D&M w/ expected payoff \$400,000



$$P(F \leq 5) = \frac{1.6}{2} = .533$$

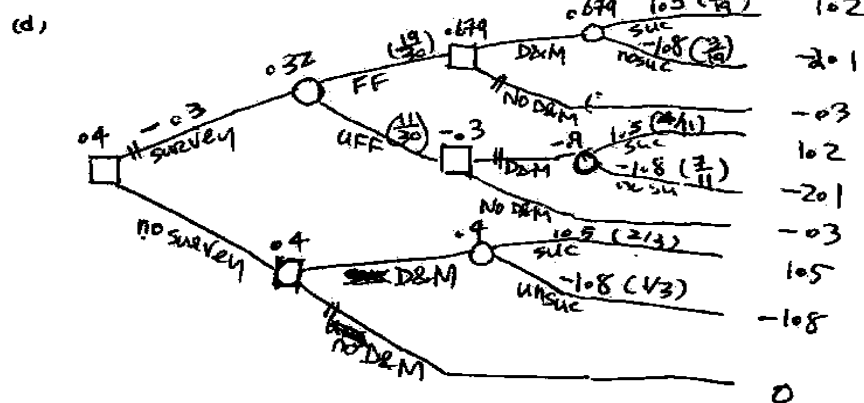
$$P(\text{UFFR}) = \frac{1}{3} = 0.333$$

$$P(\text{FF \& Uns}) = .1$$

$$P(\text{UFFE-UN5}) = .293$$

$$P(FF) = 1 - \sqrt[3]{3} = 0.633$$

$$P(KFF) = 1 \cdot \frac{1}{3} = 0.367$$



(e) Optimal policy: no survey, ~~to~~ develop & market the product.

~~Expt~~  $E(\text{survey}) = 0.32 + 0.3 = 0.62 \text{ mil}$