Math425 Test 1 Review Topics

- Method of mathematical induction, binomial expansion.
- Rational zeros theorem.
- Proof of triangle inequality.
- Definitions of $\max, \min, \sup, \inf$ of subsets of $\mathbb{R}$. Proof of $\inf S = -\sup(-S)$.
- Completeness Axiom, statement of. Proofs of Archimediean Property, denseness of rational $\mathbb{Q}$, construction of convergent subsequence of $\mathbb{Q}$ to any given real number.
- Definitions of limits, including $\pm\infty$ cases. Show limits by definition.
- Proofs of limit theorems (rules of sum, product, division, etc, including the cases for $\pm\infty$.) Property that convergence $\implies$ boundedness.
- Monotone sequences always have limits, both bounded and unbounded cases, proofs of.
- Definitions of $\limsup, \liminf$, Cauchy sequences, properties of Cauchy sequences.
- Equivalence definitions of limits and proofs: $\lim s_n = s \iff \liminf s_n = \limsup s_n = \lim s_n \iff (s_n)$ is Cauchy in the case of $\lim s_n = s \in \mathbb{R}$.
- Subsequences: Convergence $\implies$ subsequential convergence to the same limit; $\exists$ convergent subsequences to $\limsup s_n, \liminf s_n$.
- Bolzano-Weierstrass Theorem, proof of: Every bounded sequences has convergent subsequence.
- Properties of subsequential limit set S and proofs of: $S \neq \emptyset$; $\liminf s_n = \inf S, \limsup s_n = \sup S$; S is closed: i.e. $t_n \in S$ and $t_n \to t$ than $t \in S$; rules or counterexamples to sum, product, division involving $\limsup, \liminf$.
- Proof of $\liminf \left| \frac{s_{n+1}}{s_n} \right| \leq \liminf |s_n|^{1/n} \leq \limsup |s_n|^{1/n} \leq \limsup \left| \frac{s_{n+1}}{s_n} \right|$.
- Definition of convergent infinite series, Cauchy criterion.
- Comparison test, absolute convergence implies convergence, applications and proofs of.
- Ratio Test, Root Test, Integral Test, Alternating Series Test, applications and proofs of.