Oct. 2, 2002	MATH 425 Exam I	Fall Semester, 2002

Name:____________________ Score:______________

1(15pts) (i) State the Completeness Axiom

(ii) State the definitions of the following

(a) $\sup S$.

(b) $\limsup s_n$.

(c) A sequence (s_n) is Cauchy.

(d) A series $\sum a_n$ converges.

2(15pts) Prove the limit theorem of product: if $\lim s_n = s \in \mathbb{R}$ and $\lim t_n = t \in \mathbb{R}$, then $\lim s_n t_n = \lim s_n \lim t_n$. (You can use the result that convergent sequences are bounded.)

3(15pts) Prove that if (s_n) is a nondecreasing sequence then $\lim s_n$ exists and find the limit. (Warning: Don't use the result to prove itself.)

4(15pts) Assume $\limsup s_n \in \mathbb{R}$. Prove that there is a subsequence (s_{n_k}) such that $\lim_{k\to\infty} s_{n_k} = \limsup s_n$. (Warning: Don't use the result to prove itself.)

5(15pts) (a) Use the definition to show $\lim \frac{2n^2+n}{n^2-1} = 2$.

(b) Determine whether or not $\sum_{n=2}^{\infty} \frac{1}{\ln n}$ converges. State the test used and demonstrate how it is used.

(c) Determine whether or not $\sum_{n=0}^{\infty} \frac{a^n}{n!}$ converges. State the test used and demonstrate how it is used.

6(15pts) Suppose $|a_n| \le b_n$ for all $n \in \mathbb{N}$ and $\sum b_n < \infty$ converges. Prove $\sum a_n$ converges by the Cauchy criterion.

7(15pts) Prove that if (s_n) is a Cauchy sequence, then $\limsup s_n = \liminf s_n$. (Warning: Don't use the result to prove itself. In particular, you are not allowed to use the result that (s_n) Cauchy implies $\lim s_n \in \mathbb{R}$ exists.)

The End