1(15pts) (i) State the Completeness Axiom
(ii) State the definitions of the following
   (a) sup S.
   (b) lim sup s_n.
   (c) A sequence (s_n) is Cauchy.
   (d) A series $\sum a_n$ converges.

2(15pts) Prove the limit theorem of product: if $\lim s_n = s \in \mathbb{R}$ and $\lim t_n = t \in \mathbb{R}$, then $\lim s_n t_n = \lim s_n \lim t_n$. (You can use the result that convergent sequences are bounded.)

3(15pts) Prove that if $(s_n)$ is a nondecreasing sequence then $\lim s_n$ exists and find the limit. (Warning: Don’t use the result to prove itself.)

4(15pts) Assume $\lim sup s_n \in \mathbb{R}$. Prove that there is a subsequence $(s_{n_k})$ such that $\lim_{k \to \infty} s_{n_k} = \lim sup s_n$. (Warning: Don’t use the result to prove itself.)

5(15pts) (a) Use the definition to show $\lim \frac{2n^2 + n}{n^2 + 1} = 2$.
   (b) Determine whether or not $\sum_{n=2}^{\infty} \frac{1}{mn}$ converges. State the test used and demonstrate how it is used.
   (c) Determine whether or not $\sum_{n=0}^{\infty} \frac{n^2}{m^n}$ converges. State the test used and demonstrate how it is used.

6(15pts) Suppose $|a_n| \leq b_n$ for all $n \in \mathbb{N}$ and $\sum b_n < \infty$ converges. Prove $\sum a_n$ converges by the Cauchy criterion.

7(15pts) Prove that if $(s_n)$ is a Cauchy sequence, then $\lim sup s_n = \lim inf s_n$. (Warning: Don’t use the result to prove itself. In particular, you are not allowed to use the result that $(s_n)$ Cauchy implies $\lim s_n \in \mathbb{R}$ exists.)

The End