Math 314 Summer '12 Exam 1

Name: ______________________ **Score:** ______________________

Instructions: You must show supporting work to receive full and partial credits. No text book, notes, or formula sheets allowed.

1. (10 pts) True/False. For each of the following statements, please *circle* T (True) or F (False). You do not need to justify your answer. (Recall: an $m \times n$ matrix is one that has m rows and n columns.)

   (a) T or F? If A is an $m \times n$ matrix, then $\text{rank}(A) + \text{nullity}(A) = m$.

   (b) T or F? If vectors $\mathbf{v}_1, ..., \mathbf{v}_n$ are linearly dependent and $n \geq 3$, then there must be a vector that is the scalar multiple of another vector.

   (c) T or F? If $n \times n$ matrixes A and B are invertible, then they must be row equivalent to each other.

   (d) T or F? If $n \times n$ matrixes A and B are invertible, then $A + B$ is also invertible.

   (e) T or F? The column space $\text{col}(A)$ is preserved by elementary row operations on A. That is, if $A = [\mathbf{a}_1, ..., \mathbf{a}_n]$ and $\text{REF}(A) = [\mathbf{u}_1, ..., \mathbf{u}_n]$, then $\text{span}\{\mathbf{a}_1, ..., \mathbf{a}_n\} = \text{span}\{\mathbf{u}_1, ..., \mathbf{u}_n\}$.

   (f) T or F? The dimension of $\text{span}(\{\mathbf{v}_1, ..., \mathbf{v}_n\})$ is always n.

   (g) T or F? A square matrix A is invertible if $\text{nullity}(A) = \text{rank}(A)$.

2. (10 pts) Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ a & b & 3a \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$. Find all values of a and b so that $A\mathbf{x} = \mathbf{b}$ is consistent.

3. (15 pts) (a) Use elementary row reduction to find the inverse of the matrix:

$$A = \begin{bmatrix} 0 & 1 & -1 \\ 5 & 1 & 3 \\ 1 & 0 & 1 \end{bmatrix}$$

(b) Write the A as a product of elementary matrixes.

4. (10 pts) Balance the following chemical reaction: $H_3O + CaCO_3 \rightarrow H_2O + Ca + CO_2$.

5. (20 pts) The following matrices are row-equivalent:

$$A = \begin{bmatrix} 1 & 2 & 3 & -5 \\ 0 & 1 & 1 & 3 \\ 2 & -1 & 1 & 8 \\ -2 & 2 & 0 & 4 \\ 3 & 0 & 3 & 2 \end{bmatrix} \qquad R = \begin{bmatrix} 1 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 2 & 2 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Answer the following questions and justify your answer:

(a) Find the reduced row echelon form, RREF(A), of A.

(b) Find a basis for the column space, col(A), of A.

(c) Express each non-pivot column of A as a linear combination of the pivot columns.

(d) Find a basis for the null space, null(A), of A.

6. (10 pts) (a) Use row expansion only to find the determinant of $A = \begin{bmatrix} 0 & 1 & -1 \\ 5 & 1 & 3 \\ 1 & 0 & 1 \end{bmatrix}$

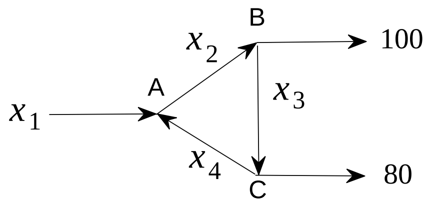
(b) Use elementary row operations only to find the determinant of A.

7. (10 pts) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation that first project a vector $\mathbf{x}$ to the x-axis and then rotate it counterclockwise 45°.

(a) Find its standard matrix $[T]$

(b) Use the matrix multiplication $T\mathbf{x} = [T]\mathbf{x}$ to find $T\left(\begin{bmatrix} 2 \\ 1 \end{bmatrix}\right)$.

8. (10 pts) Find the general flow pattern of the network s nonnegative, what is the smallest possible value for x_4?



9. (5 pts) Let $A = \begin{bmatrix} A_1 \\ \vdots \\ B_i + C_i \\ \vdots \\ A_n \end{bmatrix}$ be an $n \times n$ matrix whose ith row is the sum of two row vectors B_i and C_i. Let $B = \begin{bmatrix} A_1 \\ \vdots \\ B_i \\ \vdots \\ A_n \end{bmatrix}$ and $C = \begin{bmatrix} A_1 \\ \vdots \\ C_i \\ \vdots \\ A_n \end{bmatrix}$ be the same matrixes as A except that the ith row is B_i and C_i respectively. Prove $\det(A) = \det(B) + \det(C)$.

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2 Bonus Points: The opening day of the 2012 Summer Olympics is (a) July 4, (b) July 27, (c) August 1, (d) August 8.