Math 314 Spring 2016 **Exam 1**

Print Your Name Legibly: _____ **Score:** _____

Instructions: You must show supporting work to receive full and partial credits. No textbook, lecture notes, or formula sheets are allowed.

1. (15 pts) True/False. For each of the following statements, please *circle* T (True) or F (False). You do not need to justify your answer. (Recall: an $m \times n$ matrix is one that has m rows and n columns.)

   (a) T or F? For matrix multiplication, if $AB = 0$, then either $A = 0$ or $B = 0$.

   (b) T or F? A square matrix A is invertible then the column vectors of A is dependent.

   (c) T or F? Not every linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^m$ is a matrix transformation.

   (d) T or F? Let A be an $n \times n$ matrix. If the columns of A do not span $\mathbb{R}^n$, then the row echelon form of A must have a row of all zeros.

   (e) T or F? Every system of three equations in two unknowns always has a solution.

2. (15 pts) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation that first rotate a vector counterclockwise $90°$ and then reflect it about the x-axis.

   (a) Find its standard matrix $[T] = [T(\mathbf{e}_1), T(\mathbf{e}_2)]$. Show your work.

   (b) Find $T(\mathbf{x})$ of the vector $\mathbf{x} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$.

(*Page 1. Continue on Next Page ...*)

3. (25 pts) Matrix $A = \begin{bmatrix} 1 & -1 & * & -1 \\ 1 & 1 & 7 & 3 \\ 2 & 1 & * & 4 \end{bmatrix}$ is row equivalent to matrix $B = \begin{bmatrix} 0 & 2 & 8 & 4 \\ 1 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

(a) Find the reduced row echelon form of A, $\text{rref}(A)$.

(b) Find a largest set of linearly independent column vectors of A.

(c) Find the solution to $A\mathbf{x} = \mathbf{0}$ and express it in vector form.

(d) Write column 3 of A as a linear combination of the other columns of A and write it out in its components.

(*Page 2. Continue on Next Page ...*)

4. (13 pts) Use elementary row reduction to find the inverse A^{-1} if $A = \begin{bmatrix} 0 & 2 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

5. (12 pts) Find the LU-factorization of matrix $A = \begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix}$.

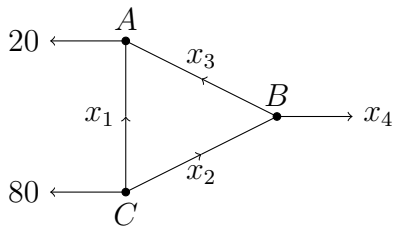
(*Page 3. Continue on Next Page ...*)

6. (10 pts) Set up a balance equation only for the following chemical reaction:

$$H_2O + CH_4 \rightarrow H_2 + CO$$

Do not solve for the solution.

7. (10 pts) A traffic network is give below, set up a system of equations for the traffic flow variables x_1, x_2, x_3, x_4. **Do not solve for the solution.**



2 Bonus Points: True or False: The Nebraska State Book is the Bible. (... The End)