**Math 314 Spring 2016 Final Exam**

**Print Your Name Legibly:**_____ **Score:** _____

**Instructions:** You must show supporting work to receive full and partial credits. No text book, notes, or formula sheets allowed.

1. (30 pts) For each of the following statements, please *circle* T (True) or F (False). You do not need to justify your answer.

   (a) T or F? The eigenvalue of a matrix is always a real number.

   (b) T or F? The eigenvalue of a symmetric matrix is always positive.

   (c) T or F? The singular value of a matrix can be any real number.

   (d) T or F? The number of singular values of a matrix equals the dimension of the null space of the matrix.

   (e) T or F? There are matrixes which do not have any eigenvalue.

   (f) T or F? Any eigenvector of a matrix is in the column space of the matrix.

   (g) T or F? For matrix multiplication it is always true that $AB = BA$.

2. (20 pts) Let $A = \begin{bmatrix} 4 & 1 \\ 2 & 0 \end{bmatrix}$.

   (a) Use elementary row reduction to find the inverse $A^{-1}$.

   (b) Find all eigenvalues of $A$.

3. (20 pts) Let $A = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$.

(a) Is $\mathbf{v} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ an eigenvector of $A$? Justify your answer.

(b) Is 1 an eigenvalue of $A$? Justify your answer.

4. (20 pts) Let $W = \text{Span}\{ \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ 6 \\ -3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix} \}$.

(a) Find a basis for $W$.

(b) Write every non-basis vector of the spanning set as a linear combination of the basis vectors.

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5. (15 pts) Let $\mathbf{u} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 0 \\ 0 \\ 2 \\ 0 \end{bmatrix}$.

   (a) Find the distance between $\mathbf{u}$ and $\mathbf{v}$.

   (b) Find the angle between $\mathbf{u}$ and $\mathbf{v}$.

6. (20 pts) Let $W = \text{span}\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \}$.

   (a) Find a basis for the orthogonal complement $W^{\perp}$ of $W$.

   (b) Find an orthogonal basis for $W^{\perp}$.

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7. (20 pts) Let matrix $A$ be diagonalized as below

$$A = \begin{bmatrix} 0 & -1 & 0 & 1 \\ 1 & 1 & -1 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 1 \\ 1 & 1 & -1 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}^{-1}$$

(a) What are the eigenvalues of $A$?

(b) Find a basis for the eigenspace of the largest eigenvalue.

(c) Find the determinant of $A$.

8. (20 pts) Find an orthogonal diagonalization (or spectral decomposition) of $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$.

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9. (15 pts) The singular value decomposition of a $2 \times 3$ matrix is given as

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \sqrt{2} & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \end{bmatrix}$$

(a) What are the eigenvalues of the product $A^TA$?

(b) What is a singular value decomposition of the transpose $A^T$?

10. (20 pts) Find a singular value decomposition of $A = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$.

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**2 Bonus Points**: True or False: Omaha used to be the capital of Nebraska. (*Page 5. ... The End*)