Name:______________________ PIN(in any 4 digits):__________ Score:__________

Instructions: You must show supporting work to receive full and partial credits. No text book, notes, formula sheets allowed.

1(10pts) (a) Is $(3t - t^3 + 1)^{1/3}$ the solution to this IVP: $\frac{dx}{dt} = \frac{1-t^2}{x}$, $x(0) = 1$? Verify your answer.

(b) Determine whether or not $x^2y^3 - 2x^3y^2 = C$ defines an implicity solution to the equation $(2y^2 - 6xy)\frac{dy}{dx} + (3xy - 4x^2) = 0$.

2(10pts) Find a general solution to the equation $y\frac{dy}{dx} = x^3 e^{-y^2}$.

3(20pts) Consider the linear equation $\frac{dx}{dt} = \frac{x}{t+1} + 2t^2 - 2$.

(a) Find a general solution to the homogeneous equation.

(b) Find a particular solution to the nonhomogeneous equation.

(c) Find the solution that also satisfies the initial condition $x(0) = 1$.

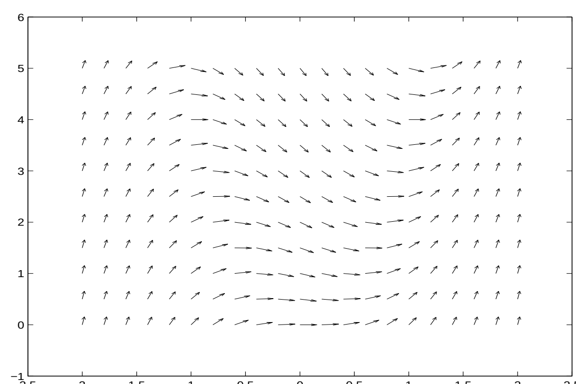
4(15pts) Does the Existence and Uniqueness Theorem apply to this initial value problem $\frac{dy}{dx} = xy^{1/2}, y(1) = 0$? If not, why not? And if not, find at least 2 solutions.

5(15pts) Consider the IVP: $x' = \frac{x^2 + t}{x}$, $x(0) = -1$.

(a) Use a step size $h = 0.25$ and the Euler method to approximate the solution in the interval $[0, 1]$.

(b) Sketch your approximating solution.

6(15pts) (a) The vector field of an equation is given below. Sketch solutions that go through these points: (i) $(-2, 0)$, (ii) $(0, 5)$.



(b) Consider the equation $x' = -x + t$. Use the isocline method to sketch a solution portrait of the equation (i.e. a few isoclines of your choice, vector field on the isoclines, and a few typical solution curves). Based on your analysis, what can you conclude about the limit $\lim_{t\to\infty} x(t)$ for any solution $x(t)$? If you concluded that $\lim_{t\to\infty} x(t) = \infty$, would $x(t)$ tend to infinity faster than t^2 as $t \to \infty$? Justify your answer.

7(15pts) Consider the autonomous equation $\frac{dx}{dt} = f(x) = x^4 - 3x^3 + 2x^2$.

(a) Sketch the phase line and classify each equilibrium point as sink, source, or node,

(b) Let $x(t)$ be a solution satisfying $x(1) = 0.5$. Then what are the limits of $\lim_{t\to\infty} x(t)$ and $\lim_{t\to-\infty} x(t)$?

END

Sdn. Key Math 221 Test 1 Summer(2), 02

1 (a) $x(t) = (3t - t^3 + 1)^{1/3}$, $x(0) = 1$, $x'(t) = \frac{1}{3}(3t - t^3 + 1)^{-2/3}(3 - 3t^2) = \frac{1-t^2}{x(t)^2} \neq \frac{1-t^2}{x}$. No

(10pts) (b) $x^2y^3 - 2x^3y^2 = C$. $\frac{d}{dx}(x^2y^3 - 2x^3y^2) = 0 \Rightarrow 2xy^3 + 3x^2y^2\frac{dy}{dx} - 6x^2y^2 - 4x^3y\frac{dy}{dx} = 0$

$\Rightarrow (3x^2y^2 - 4x^3y)\frac{dy}{dx} + (2xy^3 - 6x^2y^2) = 0$. It is Not $(2y^2 - 6xy)\frac{dy}{dx} + (3xy - 4x^2) = 0$

2 (10pts) $y\frac{dy}{dx} = x^3e^{-y^2}$ Separable $\Rightarrow ye^{y^2}dy = x^3dx \Rightarrow \frac{1}{2}e^{y^2} = \frac{1}{4}x^4 + C$.

or $\Rightarrow y = \pm\sqrt{\ln(\frac{x^4}{2} + C)}$

3 (20pts) $\frac{dx}{dt} = \frac{x}{t+1} + 2t^2 - 2$. (a) $\frac{dx}{dt} = \frac{x}{t+1}$, $x(t) = Ce^{\int\frac{1}{t+1}dt} = Ce^{\ln(t+1)} = C(t+1)$

(b) $x_p(t) = C(t)(t+1)$, $x_p'(t) = C'(t+1) + C(t) = \frac{x_p(t)}{t+1} + 2t^2 - 2 = C(t) + 2t^2 - 2$.

$\Rightarrow C'(t) = \frac{2t^2-2}{t+1} = \frac{2(t+1)(t-1)}{t+1} = 2t - 2$, $\Rightarrow C(t) = t^2 - 2t$ (or $x_p(t) = (t-1)^2$)

(c) General soln. $x(t) = C(t+1) + (t^2 - 2t)(t+1)$, $1 = x(0) = C$. $\Rightarrow x(t) = (t^2 - 2t + 1)(t+1)$ $= (t-1)^2(t+1)$

4 (15 pts): $\frac{dy}{dx} = xy^{1/2}$, $y(1) = 0$. $f(x,y) = xy^{1/2}$, $\frac{\partial f}{\partial y} = \frac{1}{2}\frac{x}{y^{1/2}}$ not continuous in any box containing the initial point $(1, 0)$. Theorem 1 does not apply.

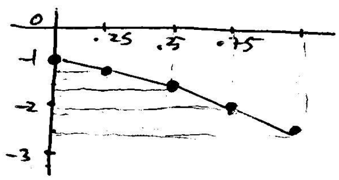
$y(x) \equiv 0$ is one solution. To find another, use separable method (optional

[$y^{-1/2}dy = xdx$ $2y^{1/2} = \frac{1}{2}x^2 + C$. $y^{1/2} = \frac{1}{4}x^2 + C$. $y(1) = 0 \Rightarrow C = -\frac{1}{4}$] and

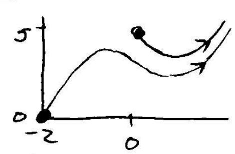
$y(x) = (\frac{1}{4}x^2 - \frac{1}{4})^2$ is a second solution.

5 (15pts). $h = 0.25$, $t_0 = 0$, $x_0 = -1$, $x_{k+1} = x_k + hf(t_k, x_k) = x_k + 0.25\frac{x_k^2 + t_k}{x_k}$ $(= x_k + h\dot{x}_k)$

k	t_k	x_k	$\dot{x}_k = \frac{x_k^2 + t_k}{x_k}$
0	0	−1	−1
1	.25	−1.25	$(-1.25)^2 + .25)/-1.25 = -1.45$
2	.5	−1.6125	−1.7675
3	.75	−2.0544	−2.176
4	1	−2.598	—



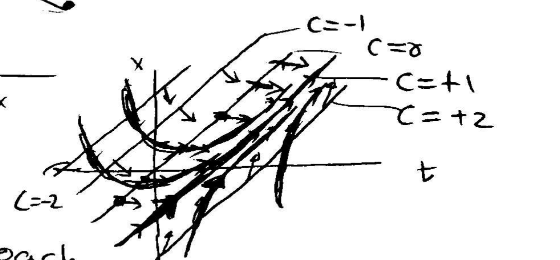
6 (15pts) (a)



(b) $x' = -x + t$

$-x + t = C$

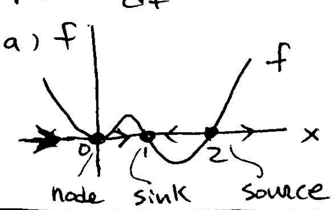
$C = 0, \pm1, \pm2, \cdots$



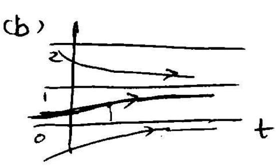
$x = t - 1$ is a solution and all solutions $x(t)$ approach it asymptotically as $t \to \infty$. Yes, $\lim_{t\to\infty} x(t) = \infty$. No, since $x(t) \approx t - 1$ as $t \to \infty$.

7 (15pts) $\frac{dx}{dt} = x^4 - 3x^3 + 2x^2 = x^2(x^2 - 3x + 2) = x^2(x-2)(x-1) = f(x)$

(a)



(b)



$\lim_{t\to\infty} x(t) = 1$
$\lim_{t\to-\infty} x(t) = 0$
if $x(0) = 0.5$

End