June 21, 2002 **Math 221 Test 2** **Summer 2002**

Name:_____ Score:_____

Instructions: You must show supporting work to receive full and partial credits. No text book, notes, formula sheets allowed.

1(15pts) Find a particular solution (synchronous) solution to $y'' + 3y' + 9y = \cos 3t$.

2(18pts) An object of mass 5 kg is given an initial upward velocity of 50 m/sec and then allowed to fall under the influence of gravity. Assume the force in newtons due to air resistance is $-10v$, where v is the velocity of the object in m/sec. Determine the time it takes to reach the maximum height.

3(17pts) A cold beer initially at 35°F warms up to 40°F in 3 min while sitting in a room of temperature 70°F. How warm will the beer be if left out for 20 min?

4(20pts) A brine solution of salt flows at a constant rate of 3 L/min into a large tank that initially held 100 L of pure water. The solution inside the tank is kept well stirred and flows out at a rate of 4 L/min. If the concentration of salt in the brine entering the tank is 0.2 kg/L, write down an initial value problem for the amount of salt inside the tank during the period before the tank runs empty and determine the first time when the tank runs empty.

5(15pts) In 1970 the population of alligators on the Kennedy Space Center grounds was estimated to be 300. In 1980 the population had grown to an estimated 1500. Use the Malthusian law (exponential growth) to estimate the alligator population in the year of 2004.

6(15pts) Find the solution to the initial value problem of an undamped, unforced mass-spring problem

$$x'' + 9x = 0, \quad x(0) = 1, x'(0) = 1.$$

If the mass is 1 kg in this system, what is the spring constant assuming the displacement in x is in meters?

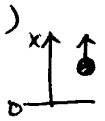
END

Math 221 Test 2 Solution Key Summer 2002

1 (15 pts) $y''+3y'+9y=\cos 3t$, $y(t)=A\cos 3t+B\sin 3t$, $y'(t)=-3A\sin 3t+3B\cos 3t$

$y''=-9A\cos 3t-9B\sin 3t \Rightarrow y''+3y'+9y=(-9A+9B+9A)\cos 3t+(-9B-9A+9B)\sin 3t$

$=9B\cos 3t-9A\sin 3t=\cos 3t \Rightarrow B=\frac{1}{9}$, $A=0$ $\boxed{y_p(t)=\frac{1}{9}\sin 3t}$

2 (18 pts) 

$m\frac{dv}{dt}=-mg-10v$, $m=5$. $\begin{cases} v'=-9.81-2v \\ v(0)=50 \end{cases} \Rightarrow v(t)=-\frac{9.81}{2}+\left(\frac{9.81}{2}+50\right)e^{-2t}$

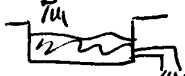
Time at max. height: $0=v(t)=-\frac{9.81}{2}+\left(\frac{9.81}{2}+50\right)e^{-2t}=0 \Rightarrow \boxed{t=-\frac{1}{2}\ln\frac{4.905}{54.905}=1.25\text{ sec}}$

(If the axis is chosen downward, then $v(t)=\frac{9.81}{2}-\left(\frac{9.81}{2}+50\right)e^{-2t}$)

3 (17 pts) By Newton's law of cooling $\frac{dT}{dt}=k(70-T)$, $T(0)=35^\circ F$

$T(t)=70+(35-70)e^{-kt}=70-35e^{-kt}$. Since $40=T(3)=70-35e^{-k\cdot 3}$

$\Rightarrow -30=-35e^{-k3}$ $\frac{30}{35}=\frac{6}{7}=e^{-k\cdot 3}$ $k=-\frac{1}{3}\ln\frac{6}{7}$. $T(20)=70-35e^{+\frac{1}{3}(\ln\frac{6}{7})20}\approx \boxed{57.5^\circ F}$

4 (20 pts) 

Volume: $V(t)=100-t$, Empty time: $V(t)=0 \Rightarrow t=100$ min.

$x(t)$ — salt in kg at t min.

$\frac{dx}{dt}=3\text{ L/m}\cdot 0.2\text{ kg/L}-4\text{ L/m}\cdot\frac{x(t)}{100-t}$, $x(0)=0$

$\Rightarrow \begin{cases} x'(t)=0.6-\frac{4x(t)}{100-t} \\ x(0)=0 \end{cases}$ for $0\le t\le 100$.

5 (15 pts) $\frac{dP(t)}{dt}=kP(t)$, $P(t)=P_0e^{kt}=300e^{kt}$ with $P(0)=300(=P_0)$

$1500=P(10)=300e^{k10} \Rightarrow k=\frac{1}{10}\ln 5 \Rightarrow P(34)=300e^{(\frac{1}{10}\ln 5)34}=\boxed{1,387}$

6 (15 pts) $x''+9x=0$. $x(t)=A\cos 3t+B\sin 3t$, $1=x(0)=A(1)+B(0)$. $1=x'(0)$

$=(-3A\sin 3t+3B\cos 3t)\big|_{t=0}=-3A(0)+3B(1) \Rightarrow A=1$, $B=\frac{1}{3}$.

$\boxed{x(t)=\cos 3t+\frac{1}{3}\sin 3t}$.

If $m=1$, then $\boxed{k=9}$ newtons/meter since $mx''=-kx$ which is $x''+9x=0$

End.