

Print Your Name Legibly:\_\_\_\_\_

Score:\_\_\_\_\_

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**Instructions:** You must show supporting work to receive full and partial credits. No text book, notes, formula sheets allowed.

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1(**10pts**) Use the definition of Laplace transform to find  $\mathcal{L}\{f(t)\}(s)$  if  $f(t) = \begin{cases} 0, & t < 2 \\ 2, & t \geq 2. \end{cases}$

2(**20pts**) Find the Laplace transform or inverse transform:

(a)  $\mathcal{L}\{te^{2t}f(t)\}(s)$  given that  $\mathcal{L}\{f(t)\}(s) = \frac{1}{\sqrt{s}}$ .

(b)  $\mathcal{L}^{-1}\left\{\frac{5-2s}{s(s^2-2s+5)}\right\}(t)$ .

3(**15pts**) Use the Laplace method to solve the initial value problem:  $x'' + x' = t$ ,  $x(0) = 0$ ,  $x'(0) = -1$ .

4(**20pts**) Find two linearly independent eigensolutions and the general solution for the system of equations

$$\begin{cases} x' = x + 2y \\ y' = 4x + 3y. \end{cases}$$

5(**15pts**) Use the Laplace method to find the solution to the integral equation:  $f(t) + \int_0^t (t - \tau)f(\tau)d\tau = 1$ .

6(**20pts**) The eigenvalues for the matrix  $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$  are given as 0, 1, 2. Find the general solution to the system of equations  $\mathbf{x}' = A\mathbf{x}$ .