

Print Name:_____

Score:_____

Instructions: You must show supporting work to receive full and partial credits. No text book, notes, formula sheets allowed.

1(20pts) (a) Find a general solution to $y''' - 3y'' + 2y' = 0$.

(b) Find the particular solution with the initial conditions $y(0) = 1, y'(0) = 2, y''(0) = 4$.

2(15pts) The linear differential equation of real-valued constant coefficients

$$a_3y^{(3)} + a_2y'' + a_1y' + a_0y = \sin x + 3xe^{4x}$$

has $y_1 = e^{4x}, y_2 = \cos x$ as solutions to the homogeneous equation. Find a **FORM** for a particular solution to the nonhomogeneous equation.

3(15pts) Use the method of undetermined coefficients to find a particular solution to the equation $y' + 3y = e^{-3x}$.

4(15pts) Use a reduction of order to find a second solution to this equation $x^2y'' - 4xy' + 6y = 0$ given that $y_1(x) = x^2$ is a solution.

5(15pts) Use the method of variation of parameters to find a particular solution to the equation $y'' + y = \cos(x)$ given that $y_1(x) = \cos(x)$, $y_2(x) = \sin(x)$ are solutions to the homogeneous equation.

6(20pts) A force of 400 newtons stretches a spring 2 meters. A mass of 50 kilograms is attached to the end of the spring inside a vacuum chamber and is initially released 3 m below the equilibrium position with an upward velocity of 8 m/s. Find the equation of the motion, and express the solution in the amplitude-phase-angle form.