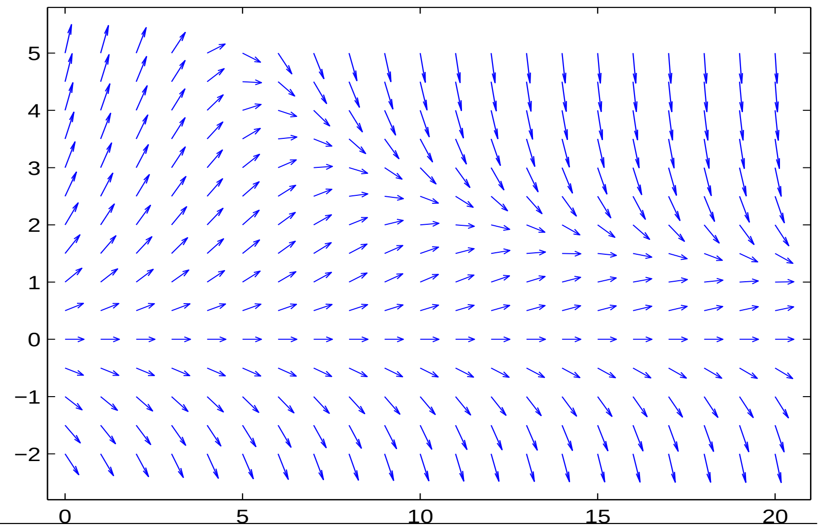
Name:______________________________ Score:______________________

Instructions: You must show supporting work to receive full and partial credits. No text book, notes, formula sheets allowed.

1(10pts) The vector field of an equation is given in figure. Sketch solutions with these initial values: (i) $y(5) = 1$, (ii) $y(10) = 5$, (iii) $y(3) = 0$.



2(15pts) Use Euler's method to approximate the solution to the IVP: $y' = 2x - y^2,\ y(0) = 2$ in the interval $[0, 1]$ by discretizing the interval into 4 equal parts. Sketch your approximating solution.

3(15pts) Consider the autonomous equation $\dfrac{dx}{dt} = x^2 - x - 2$

(a) Sketch the graph of the equation and sketch the phase line.

(b) Classify the stability of all equilibrium solutions.

(c) Sketch a solution portrait of the equation, and state the limit of $\lim_{t\to\infty} x(t)$ for the solution with the initial condition $x(1) = 0$.

4(15pts) Find a general solution to the equation $(1 + x^2)\dfrac{dy}{dx} = 2y^2$.

5(15pts) Find a general solution to the equation $y' + \dfrac{2}{1+x}y = \dfrac{1}{1+x}$.

6(15pts) Find a general solution to the exact equation $(3yx^2 + 2x + 1)dx + (x^3 + 2y)dy = 0$.

7(15pts) (a) A tank initially contains 60 gallon of pure water. Salt water containing 1 lb of salt per gallon enters the tank at 2 gal/min. and the mixed solution leaves the tank at the same incoming rate. Derive a differential equation with initial condition for the amount salt inside tank at any time t. **Do not solve the equation**.

(b) Let $P(t)$ denote the population of a species at time t. Assume the per-capita birth rate is a constant and the per-capita death rate is proportional to the square root of the population. Derive a differential equation model for the population. **Do not solve the equation**.

END