Math 221-02 Test 3 **Spring 2008**

Name:_____ Score:_____

Instructions: You must show supporting work to receive full and partial credits. No text book, personal notes, formula sheets allowed. **One formula sheet is provided on next page.**

1(**20pts**) Find the general solution of the equations

$$\begin{cases} x' = 5x - 5y \\ y' = 2x - y \end{cases}$$

2(**20pts**) Find the general solution of the equations

$$\begin{cases} x' = 2x - 5y \\ y' = 2y \end{cases}$$

3(**20pts**) Use definition *only* to find the Laplace transform of the function

$$f(t) = \begin{cases} t, & t < 1 \\ 0, & t \geq 1 \end{cases}$$

(You can use the formula $\int te^{bt}dt = \frac{1}{b}te^{bt} - \frac{1}{b^2}e^{bt} + C$.)

4(**20pts**) Find the Laplace transform or inverse transform:

(a) Find $\mathcal{L}\{te^{2t}\sin t\}(s)$.

(b) If $\mathcal{L}^{-1}\{\frac{2}{s(s^2+2s+2)}\}(t)$.

5(**20pts**) Use the Laplace method to solve the initial value problem

$$x'' + 4x = 4,\ x(0) = 4,\ x'(0) = 0.$$

Bonus 2pts: The state fish of Nebraska is _____

The End

Table of Laplace Transforms and Inverse Transforms

$f(t) = \mathcal{L}^{-1}\{F(s)\}(t)$	$F(s) = \mathcal{L}\{f(t)\}(s)$
$t^n e^{at}$	$\dfrac{n!}{(s-a)^{n+1}}\ , \quad s > a$
$e^{at}\sin bt$	$\dfrac{b}{(s-a)^2+b^2}\ , \quad s > a$
$e^{at}\cos bt$	$\dfrac{s-a}{(s-a)^2+b^2}\ , \quad s > a$
$e^{at}f(t)$	$F(s)\big\vert_{s\to s-a}$
$u(t-a)f(t)$	$e^{-as}\mathcal{L}\{f(t+a)\}(s)$, alternatively,
$u(t-a)\big[f(t)\big\vert_{t\to t-a}\big]$	$e^{-as}F(s)$
$\delta(t-a)f(t)$	$f(a)e^{-as}$
$f^{(n)}(t)$	$s^nF(s) - s^{n-1}f(0) - \cdots - f^{(n-1)}(0)$
$t^n f(t)$	$(-1)^n\dfrac{d^n}{ds^n}F(s)$, with special case
	$\mathcal{L}^{-1}\{F(s)\}(t) = -\dfrac{1}{t}\mathcal{L}^{-1}\{F'(s)\}(t)$
$\dfrac{f(t)}{t}$	$\displaystyle\int_s^\infty F(\sigma)\ d\sigma$, with special case
	$\mathcal{L}^{-1}\{F(s)\}(t) = t\mathcal{L}^{-1}\left\{\displaystyle\int_s^\infty F(\sigma)\ d\sigma\right\}(t)$
$\displaystyle\int_0^t f(\tau)\ d\tau$	$\dfrac{F(s)}{s}$
$f(t) * g(t)$	$F(s)G(s)$