Math 221 Test 3 **Spring 2006**

Name:_____ Score:_____

Instructions: You must show supporting work to receive full and partial credits. No text book, personal notes, formula sheets allowed. One formula sheet will be provided.

1(**20pts**) Find the general solution of the equations

$$\begin{cases} x' = 4x + 2y \\ y' = -3x - y \end{cases}$$

2(**20pts**) You are given that a system of linear differential equations $\vec{x}' = A\vec{x}$ has a complex eigenvalue $\lambda = -1 + 2i$ and a corresponding eigenvector

$$\xi = \begin{pmatrix} 3 \\ 2+i \end{pmatrix}.$$

Use the information to construct a general solution of the equations.

3(**10pts**) Use definition *only* to find the Laplace transform of the function

$$f(t) = \begin{cases} 0, & t < 1 \\ t, & t \geq 1 \end{cases}$$

Using any other method receives no credit.

4(**10pts**) Fill in the blanks and justify your answers:

(a) If $\mathcal{L}[f(t+2)](s) = \ln \dfrac{s^2}{s^2+1}$, then $\mathcal{L}[u(t-2)f(t)] =$ _____.

(b) If $\mathcal{L}^{-1}[F(s)](t) = \dfrac{\sqrt{t}}{t+1}$, then $\mathcal{L}^{-1}[F(s-2)](t) =$ _____.

5(**20pts**) Use the Laplace method to solve the initial value problem

$$x' + x = f(t),\ x(0) = 0$$

where $f(t) = \begin{cases} 0, & t < 2 \\ 2, & t \geq 2 \end{cases}$

6(**20pts**) Use the Laplace method to solve the initial value problem

$$\begin{cases} x' - y' - 6x = 0 \\ x' + 2y' - 3y = 0 \\ x(0) = 0,\ y(0) = 1. \end{cases}$$

(Your solution should be $x(t) = -e^{2t} + e^{3t},\ y(t) = 2e^{2t} - e^{3t}$.)

Bonus 2pts: The state fish of Hawaii is _____

The End