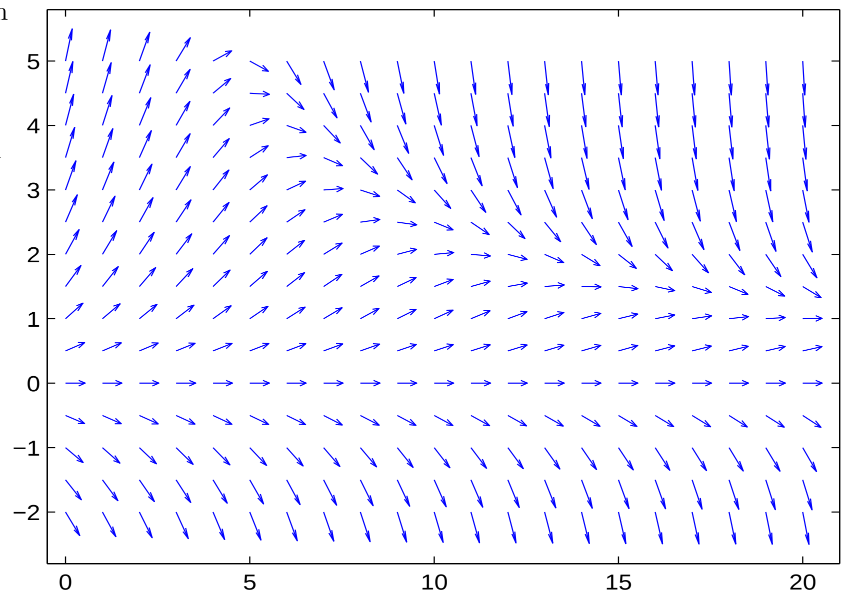
Math 221 Test 1 **Spring 2006**

Name:______________________________ Score:______________________

Instructions: You must show supporting work to receive full and partial credits. No text book, notes, formula sheets allowed.

1(15pts) (a) The vector field of an equation is given in figure. Sketch solutions with these initial values: (i) $y(0) = 0$, (ii) $y(8) = -1$, (iii) $y(4) = 0.5$.

(b) Verify if $y(x) = xe^{-2x}$ is a solution to the equation $y'' + 4y' + 4y = 0$.



2(15pts) Use Euler's method to approximate the solution to the IVP: $y' = y^2x + 1,\ y(1) = -1$ in the interval $[1, 2]$ by discretizing the interval into 4 equal parts. Sketch your approximating solution.

3(15pts) Consider the autonomous equation $\dfrac{dx}{dt} = 1 + x - x^2 - x^3$

(a) With the help of a graphical calculator, plot the graph of the right hand function of the equation over the interval $-2 < x < 2$.

(b) Classify the stability of all equilibrium solutions in the interval.

(c) Sketch a solution portrait of the equation, and state the limit of $\lim_{t\to\infty} x(t)$ for the solution with the initial condition $x(1) = 0$.

4(15pts) Find a general solution to the equation $(x + 1)y' + 2y = x$.

5(15pts) Find the solution to the initial value problem $(1 + x^2)\dfrac{dy}{dx} = 2xy^2,\ y(0) = 2$.

6(15pts) A tank initially contains 60 gallon of pure water. Brine containing 1 lb of salt per gallon enters the tank at 2 gal/min. and the mixed solution leaves the tank at the same incoming rate.

(a) Derive a differential equation with initial condition for the amount salt inside tank at any time t.

(b) What is approximately the amount of salt in the tank after a long time?

7(10pts) Let $P(t)$ denote the population of a species at time t. Assume the per-capita birth rate is a constant and the per-capita death rate is proportional to the square root of the population. Derive a differential equation model for the population. If the per-capita birth rate is 4 per year and the proportionality constant is 2 per year per square root of population, what is the carrying capacity of the population? (*Hint*: Use Phase Diagram method to justify your carrying capacity find.)

END