Math 221/821	**Final Exam**	**April 30, 2001**

Name:________________________________ **Score:**_____________

Instructions: You must show supporting works to receive full and partial credits.

1(16 pts) Find the (explicit) solution to the initial value problem of the separable equation $xy\dfrac{dy}{dx} = y^2 - 1, y(1) = -2$.

2(16 pts) Find the general solution to the 1st order linear equation $y' + 2ty = t$. (You must solve it by the method for 1st order linear equations. Any other way will receive significantly small credit.)

3(18 pts) (a) Use the Euler method to approximate the solution to the initial value problem $y' = t - y, y(0) = 0$ at these points: $t = 0, 0.1, 0.2, 0.3$. (Solution by any other method receives no credit.)
(b) Sketch your approximating solution.

4(16 pts) The characteristic equation for a homogeneous, 5th order linear, and real constant coefficient equation $ay^{(5)} + by^{(4)} + cy''' + dy'' + ey' + fy = 0$ has these roots: $1, -2, -2, 2 \pm 3i$. Find the general solution to the differential equation.

5(18 pts) Use the method of undetermined coefficients to find a suitable **form** for a particular solution $Y(t)$ to the equation $ay'' + by' + cy = 3 + 2te^{2t} - \cos(2t)$ given that $y_1(t) = 1, y_2(t) = e^t$ are two solutions to the homogeneous equation $ay'' + by' + cy = 0$. (Do not attempt to solve the coefficients!)

6(18 pts) Use the method of variation of parameters to find a particular solution to the equation $y''+by'+cy = \sin t$ given that $y_1(t) = \cos t, y_2(t) = \sin t$ are two solutions to the homogeneous equation $y'' + by' + cy = 0$. (Solution by any other method receives no points. You need to use this formula $\sin^2 t = (1-\cos(2t))/2$.)

7(18 pts) Consider a system of two competing species $x' = x(1 - x - y),\ y' = y(a - y - bx)$.
(a) Sketch a phase portrait of the system when $a = 2, b = 4$. (Make sure to include all the equilibrium points, typical direction vectors both on and off the nullclines, typical solution curves, and special solution curves such as separatrixes, if any.)
(b) If $0 < a < 1$, then which relation, $b > a$ or $0 < b < a$, will result in a co-existing equilibrium point? Give a brief justification to your answer.

8(18 pts) (a) Verify that $\sin y + xy = 1$ is an implicit solution to the differential equation

$$\frac{dy}{dx} = -\frac{y}{\cos y + x}.$$

(b) Find the general solution to $y'' = 0$.
(c) The direction field of a differential equation is given by the graph. Sketch an approximating solution with the initial point $y(0) = 2$.

9(18 pts) Find the following Inverse Laplace Transforms:

$$(a) \quad \mathcal{L}^{-1}\left\{\frac{e^{-3s}}{s^2 + 3s}\right\}(t) \qquad\qquad (b) \quad \mathcal{L}^{-1}\left\{\frac{2s}{s^2 + 4s + 13}\right\}(t)$$

10(10 pts) Use the **definition** for Laplace Transforms to find $\mathcal{L}\{u(t-2)\}(2)$, with $u(t)$ being the unit step function. (A solution by the Laplace Transformation Table receives no points!)

11(16 pts) (a) Express the function $f(t) = \begin{cases} 1 - t, & 0 \le t < 2 \\ 0, & 2 \le t \end{cases}$ in terms of a combination of unit step functions.
(b) Find the Laplace Transform, $\mathcal{L}\{f(t)\}(s)$, of the function f of (a).

12(18 pts) Find the Laplace Transform $\mathcal{L}\{y(t)\}(s)$ of the solution $y(t)$ to the initial value problem

$$y'' + 2y' + 3y = 2u(t-2)e^{2t}, \quad y(0) = 0, y'(0) = -1.$$

(Do not solve for $y(t)$.)

END