The Laplace transform is part of a package that can be introduced with
the command with(inttrans). Within this package, Maple can directly
solve simple initial value problems for equations and systems using the
command dsolve with the specification method=laplace. The solution is
displayed, but the application of the Laplace transform and of its inverse
remain hidden.

EXAMPLE 1 To solve the initial value problem $x'' - 3x' + 2x = 0$, $x(0) = 0$, $x'(0) = 1$ with the help of the Laplace transform, we
proceed as follows:

```maple
> with(inttrans):
> eq := diff(x(t),t$2)-3*diff(x(t),t) + 2*x(t) = 0;

eq := (D@@2)(x(t)) - 3*(D(x(t)) + 2*x(t) = 0

> dsolve(eq,x(t)=0,D(x)(0)=1,x(t),
method=laplace);

and Maple displays the answer,

$x(t) = e^{(2t)} - e^{t}$
```
EXAMPLE 2  To solve the initial value problem for the linear nonhomogeneous system
\[
\begin{align*}
  x' &= x - 2y - t, \quad x(0) = y(0) = 0 \\
y' &= 3x + y,
\end{align*}
\]
with the help of the Laplace transform, we proceed as follows:

\[
\begin{align*}
  &\text{with(inttrans)}: \\
  &\text{eq} := \text{diff}(x(t), t) = x(t) - 2y(t) - t, \\
  &\quad \text{diff}(y(t), t) = 3x(t) + y(t);
  \\
  &\quad eq := \frac{\partial}{\partial t} x(t) = x(t) - 2y(t) - t, \quad \frac{\partial}{\partial t} y(t) = 3x(t) + y(t)
  \\
  &\text{dsolve}([eq, x(0)=0, y(0)=0], \{x(t), y(t)\}, \text{method=laplace});
\end{align*}
\]
and Maple displays the answer,
\[
\begin{align*}
y(t) &= -\frac{3}{7} t - \frac{6}{49} + \frac{6}{49} e^t \cos(\sqrt{6}t) + \frac{5}{98} e^t \sqrt{6} \sin(\sqrt{6}t), \\
x(t) &= \frac{1}{7} t - \frac{5}{49} - \frac{2}{49} e^t \sqrt{6} \sin(\sqrt{6}t) + \frac{5}{49} e^t \cos(\sqrt{6}t)
\end{align*}
\]
Maple is also capable of directly solving initial value problems for differential equations and systems for certain discontinuous vector fields. In Maple's language the unit step function is called Heaviside, and \( u_c(t) \) is denoted by Heaviside(t-c).

EXAMPLE 3  We can solve the initial value problem \( x'' = 3x' - 2x + u_2(t), \quad x(0) = x'(0) = 0 \) as follows:

\[
\begin{align*}
  &\text{with(inttrans)}: \\
  &\text{eq} := \text{diff}(x(t), t$2$) - 3*\text{diff}(x(t), t) + 2\text{x(t)} - \text{Heaviside(t-2)} = 0; \\
  &\text{dsolve}([eq, x(0)=0, D(x)(0)=0], x(t), \text{method=laplace});
\end{align*}
\]
and Maple displays the answer,
\[
x(t) = \frac{1}{2} \text{Heaviside}(t - 2) (1 - 2e^{(t-2)} + e^{(2t-4)})
\]
where \( \text{Heaviside}(t-2) \) means \( u_2(t) \).

Remark. If Maple provides an answer involving \( e \) to a complex power, use Euler's formula,
\[
e^{it} = \cos t + i \sin t,
\]
to express the solution in terms of trigonometric functions.

In many cases Maple is unable to solve the equation directly, so you have to combine analytic methods with those provided by the computer. In such cases you can use Maple to compute the Laplace transforms of certain functions and equations and the inverse Laplace transforms of certain functions.
EXAMPLE 4 To compute the Laplace transform of the function \( x(t) = t^2 + \sin t \), proceed as follows:

\[
\text{> with(intrans):} \\
\text{> laplace}(t^2+\sin(t)=x(t), t, s);
\]

and Maple displays the answer,

\[
\frac{2}{s^3} + \frac{1}{s^2 + 1} = \text{laplace}(x(t), t, s)
\]

EXAMPLE 5 To compute the Laplace transform of the equation \( x'' - x = \sin at \), proceed as follows:

\[
\text{> with(intrans):} \\
\text{> laplace(diff(x(t), t$2$)-x(t)=sin(a$t), t, s);
\]

and Maple displays the answer

\[
s(\text{laplace}(x(t), t, s) - x(0)) - D(x)(0) - \text{laplace}(x(t), t, s) = \frac{a}{a^2 + a^2}
\]

EXAMPLE 6 To compute the inverse Laplace transform of the function

\[
\frac{1}{s - a} + \frac{s^2 - a^2}{(s^2 + a^2)^2}
\]

proceed as follows:

\[
\text{> with(intrans):} \\
\text{> invlaplace}(1/(s-a)+(s^2-a^2)/(s^2+a^2)^2, s, t);
\]

and Maple displays the answer

\[
e^{at} + t \cos(at)
\]

All these computer techniques are helpful in dealing with initial value problems that can be approached with the Laplace transform.