Math 221 Test 2 **Fall 2015**

Print Your Name:____________________ Score:____________________

Instructions: You must show supporting work to receive full and partial credits. No text book, notes, formula sheets allowed.

1(17pts) Determine the form of a particular solution for each of the equations. (Do not solve for the undetermined constants.)

(a) $y'' + 2y' + 2y = t + e^t$.

(b) $y'' + 2y' + 2y = 2e^{-t}\sin t$.

2(17pts) Consider the differential equation $y'' - y = t^2$

(a) Use the method of undetermined coefficients to find a particular solution to the equation.

(b) Find a general solution to the equation.

3(16pts) One solution to the equation $ty'' - (t+1)y' + y = 0$ is given to be $y_1(t) = e^t$. Use the method of reduction of order to find a second solution $y_2(t)$ to the equation. (Formula you may need this formula $\int te^{at}dt = te^{at}/a - e^{at}/a^2 + C$.)

4(16pts) Two solutions to the homogeneous part of the nonhomogeneous equation $t^2y'' - 2ty' + 2y = t$ are given as

$$y_1(t) = t, \qquad y_2(t) = t^2.$$

Use the method of variation of parameters to find a particular solution $y_p(t)$ to the nonhomogeneous equation.

5(17pts) Consider the matrix $A = \begin{bmatrix} 2 & 0 & -2 \\ -2 & 0 & 2 \\ 1 & 1 & 1 \end{bmatrix}$.

(a) Find the characteristic equation for A.

(b) Find all eigenvalues of A. You need to show work for your solution.

(c) Verify by definition that vector $\mathbf{v} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$ is an eigenvector of A, and identify the corresponding eigenvalue.

6(17pts) Consider the system of equations $\mathbf{x}' = A\mathbf{x}$ with $A = \begin{bmatrix} -5 & 3 \\ -6 & 4 \end{bmatrix}$.

(a) Find a general solution of the system. You must show work to justify your answer.

(b) Find the solution satisfying the initial conditions $x_1(0) = 3, x_2(0) = 4$. (No approximating values are accepted.)

2 pt Bonus Question: There are ________________ universities in the Big Ten Conference.

END