Math 221 Test 1 **Fall 2015**

Name:______________________________ Score:______________________

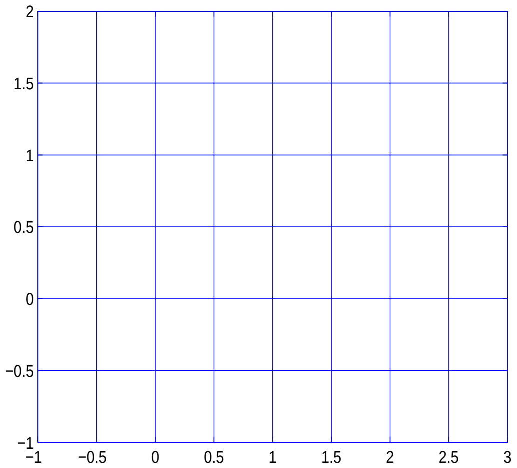
Instructions: You must show supporting work to receive full and partial credits. No text book, notes, formula sheets allowed.

1(15pts) (a) Verify if $y(x) = \arctan x + x$ is a solution to the equation: $y'' + \frac{2x}{1+x^2}y' = \frac{2x}{1+x^2}$.

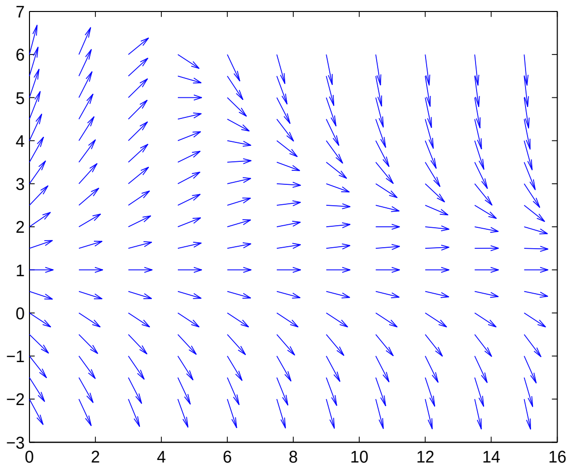
(b) Verify if $x^2 + e^{x+y^2} = 2$ defines an implicity solution to the equation $\frac{dy}{dx} = \frac{xe^x}{ye^{y^2}}$.

2(15pts) (a) For the differential equation $y' = x - y^2$, sketch its vector field in figure (a) at these points: $(0,0), (1,0), (2,0), (0,1), (1,1), (2,1)$.

(b) The vector field of an equation is given in figure (b). Sketch solutions that go through these points: (i) $(3,0)$, (ii) $(2,1)$, (iii) $(0,2)$.



(a)



(b)

3(15pts) Use Euler's method to approximate the solution to the IVP: $y' = x - 2y^2$, $y(2) = 1$ in the interval $[2,3]$ by discretizing the interval into 4 equal parts. Sketch your approximating solution.

4(30pts) Determine the type of each equation and then find the solutions. Leave your solutions in implicit form if there is no obvious way to express them as explicit solutions.

(a) $\frac{dy}{dx} = \frac{2x+xy}{1+y}$, $y(0) = -1$.

(b) $(2x^2y + \cos y)\frac{dy}{dx} = 1 - 2xy^2$.

5(15pts) At the rate of 2 liter per minute pure water is pumped into a tank containing 100 liter of brine solution with 20 kg of salt initially. The well-mixed solution flows out at the rate of 1 L/m. Assume the tank has a capacity of holding 200 L solution. Write a differential equation for the problem. When will the tank start to overflow? How many salt is in the tank 20 minutes after water starts to flow into the tank?

6(10pts) A ball of 0.5 kg is tossed up with an initial velocity of 2 m/s from 1 m above the ground. Assume the air resistance is proportional to the velocity and the proportionality is $b = 20$N-sec/m. Write a differential equation for the problem. Find the time when the ball reaches its maximal height.

2 pt Bonus Question: The University of Nebraska system was founded in the year of ________________

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