Math 221 Final Exam **Fall 2015**

Name:______________________________ Score:__________

Instructions: You must show supporting work to receive full and partial credits. No text book, personal notes, formula sheets allowed. Turn off all electronic devices except for calculators.

1(15 pts) (a) Use the Euler method to approximate the solution to the initial value problem $y' = 2t - y,\ y(1) = 0$ in the interval $1 \le t \le 2$ with step-size $h = 0.25$.

(b) Sketch your approximating solution.

2(20 pts) Solve the initial value problem $\dfrac{dy}{dx} + \dfrac{1}{x+2}y = 3x, y(0) = 1$.

3(15 pts) Determine the type of the equation $2x \sin y dx - \cos y dy = 0$, and find the general solution to the equations.

4(20 pts) Consider the system of equations

$$\begin{cases} x' = 2x - y \\ y' = 8x - y^2 \end{cases}$$

(a) Find all equilibrium solutions of the system.

(b) Classify each equilibrium solution as a saddle, a (spiral) source, or a (spiral) sink.

5(10 pts) The characteristic equation for a homogeneous linear equation of constant coefficients, $a_5y^{(5)} + a_4y^{(4)} + a_3y''' + a_2y'' + a_1y' + a_0y = 0$, is factored as $a_5r^5 + a_4r^4 + a_3r^3 + a_2r^2 + a_1r + a_0 = 2(r^3 + r)(r + 1)^2 = 0$. Find a general solution to the differential equation.

6(20 pts) Use the method of undetermined coefficients to find the form of a particular solution $y_p(x)$ to the equation $y'' - 2y' - 3y = 2e^x + 1$. You do not need to find the values of the coefficients.

7(20 pts) Use the method of variation of parameters to find a particular solution to the nonhomogeneous equation $y'' + y = \sec^3 t$.

8(10 pts) For a system of 2 equations $\mathbf{x}' = A\mathbf{x}$, it is given that A has eigenvalues $1 \pm 2i$, and that an eigenvector for $1 + 2i$ is $\begin{pmatrix} 3+i \\ 2i \end{pmatrix}$. Find a general solution to the system of equations.

9(20 pts) Use the method of eigenvalues to find the general solution to the system of differential equations

$$\begin{cases} x' = -y + z \\ y' = 14x + 5y - 19z \\ z' = 2x - 2z \end{cases}$$

10(15 pts) Find the Inverse Laplace Transform of $F(s) = \dfrac{21s + 20}{s^3 + 6s^2 + 10s}$.

11(15 pts) Find the Laplace Transform of $f(t) = u(t-1)te^{2t}\sin(3t)$.

12(20 pts) Use the method of Laplace Transform to solve the initial value problem:

$$y'' + 2y' + y = 2\delta(t-2), \quad y(0) = 0, y'(0) = 1.$$

END

$\mathcal{L}[\sin bt](s) = \dfrac{b}{s^2+b^2}$	$\mathcal{L}[e^{at}f(t)](s) = \mathcal{L}[f(t)](s-a)$	$\mathcal{L}[u(t-a)f(t-a)](s) = e^{-as}\mathcal{L}[f(t)](s)$
$\mathcal{L}[\cos bt](s) = \dfrac{s}{s^2+b^2}$	$\mathcal{L}[t^n f(t)](s) = (-1)^n \dfrac{d}{ds}\mathcal{L}[f(t)](s)$	$\mathcal{L}[\delta(t-a)f(t)](s) = e^{-as}f(a)$