Name:____________________________ Score:______________________

Instructions: You must show supporting work to receive full and partial credits. No text book, notes, formula sheets allowed.

1(**20pts**) Find all equilibrium solutions of the system of equations $\begin{cases} x' = x(3 - x - y) \\ y' = y - x - 1. \end{cases}$

2(**20pts**) Verify, not solve, that $(\bar{x}, \bar{y}) = (1, 0)$ is an equilibrium solution to this system of competing species

$$\begin{cases} x' = x - x^2 - xy \\ y' = 2y - y^2 - xy. \end{cases}$$

Use linearization to determine if it is a sink, or a source, or a saddle point.

(Continue on Next Page ...)

3(**20pts**) Find the Laplace transformations of the following functions:

(a) $f(t) = e^{-t}$, using the definition *only*.

(b) $g(t) = \begin{cases} 0, & t < 1 \\ 1, & 1 \le t < 2 \\ t, & t \ge 2 \end{cases}$, which you need to express it first as a combination of various on-off switch functions.

4(**20pts**) Find the inverse Laplace transforms of the following functions:

(a) $F(s) = \dfrac{4e^{-2s}}{s^3 + 4s}$

(b) $G(s) = \dfrac{s+1}{s^2 - 4s + 5}$

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5(**20pts**) Use the Laplace method to solve the initial value problem of the system of equations:

$$\begin{cases} x' = y \\ y' = -4x + \delta(t-\pi) \\ x(0) = 1, \ y(0) = 0. \end{cases}$$

$f(t) = \mathcal{L}^{-1}\{F(s)\}(t)$	$F(s) = \mathcal{L}\{f(t)\}(s)$
$e^{at}f(t)$	$F(s)\big\|_{s\to s-a}$
$u(t-a)f(t)$	$e^{-as}\mathcal{L}\{f(t+a)\}(s)$, alternatively,
$u(t-a)f(t-a)$	$e^{-as}F(s)$
$\delta(t-a)f(t)$	$f(a)e^{-as}$
$f^{(n)}(t)$	$s^nF(s) - s^{n-1}f(0) - \cdots - f^{(n-1)}(0)$
$t^nf(t)$	$(-1)^n\dfrac{d^n}{ds^n}F(s)$

2 pt Bonus Question: The University of Nebraska system was founded in the year of ________________ (... *The End*)