Math 221 Test 1 **Fall 2013**

Name:_____ Score:_____

Instructions: You must show supporting work to receive full and partial credits. No text book, notes, formula sheets allowed.

1(15pts) Use an appropriate substitution to transform this equation $\frac{dy}{dx} = (3x + y + 1)^2 + 1$ to a separable equation in the new variable. **DO NOT solve for the solution.**

2(20pts) Consider the autonomous equation $\frac{dx}{dt} = x^2(4 - x^2)$.

(a) With the aid of a graphical calculator or by hand, sketch the phase line in the interval $[-3, 3]$.

(b) Classify the stability of each equilibrium point in the interval.

(c) Sketch a solution portrait of the equation, including the one with the initial condition $x(1) = 3$. What is the limit of $\lim_{t\to\infty} x(t)$ of this particular solution?

3(15pts) Find a general solution to the linear equation $(x^2 - 1)\frac{dy}{dx} + 2xy = 4x$ for $x > 1$.

4(15pts) Determine the type of the equation $x\frac{dy}{dx} = (2x^2 + 1)(1 + y^2)$, and then find a general solution.

5(15pts) Verify this equation $(y + 2x)dx + (x + 3y^2)dy = 0$ is exact, and then find an implicit general solution.

6(15pts) Initially a cup of coffee was 70°C. It sat in a room of 28°C and cooled to 40°C after 10 minutes.

(a) Write an ordinary differential equation with initial condition for the temperature T as a function of time t.

(b) Assume a general solution to the equation is $T(t) = 28 + Ce^{-kt}$ (note you do not need to derive this). What are the parameter values C and k?

5 point Bonus Question: How many schools are in the Big10 Conference? (a) 10 (b) 11 (c) 12 (d) 13

END