Math 221 Practice Test 3

Name:________________________ Score:__________

Instructions: You must show supporting work to receive full and partial credits. No text book, personal notes, formula sheets allowed. One formula sheet will be provided.

1(**10pts**) For the predator-prey system below

$$\begin{cases} x' = 5x - x^2 - xy \\ y' = xy - 2y \end{cases}$$

(a) Find all equilibrium solutions of the system.

(b) Use linearization to classify the coexisting equilibrium solution as a sink, or a source, or a saddle.

(c) Use linearization also to approximate solutions of the system near the coexisting equilibrium solution.

2(**15pts**) Consider the equations

$$\begin{cases} x' = xy - 2 \\ y' = x - 2y \end{cases}$$

(a) Find all equilibrium solutions of the system.

(b) Use linearization to classify each equilibrium solution as a sink, or a source, or a saddle.

3(**10pts**) Use definition *only* to find the Laplace transform of the function

$$f(t) = \begin{cases} e^{-t}, & t < 2 \\ 0, & t \geq 2 \end{cases}$$

Using any other method receives no credit.

4(**20pts**) Find the Laplace transforms of these functions

(a) $(\sin t - 2t)^2$

(b) $f(t) = \begin{cases} t, & t < 2 \\ 1, & 2 \leq t < 3, \\ 0, & t \geq 3 \end{cases}$ which must be first expressed in terms of unit step functions.

5(**20pts**) Find the Laplace inverses of these functions

(a) $\dfrac{(s+1)e^{-3s}}{s^2 + 4s + 8}$

(b) $\dfrac{s^2 - 5s + 3}{(s+1)(s^2 - 4s + 4)}$

6(**15pts**) Solve the initial value problem: $\begin{cases} x'' + 2x' + 2x = 2\delta(t-2) \\ x(0) = 1, \ x'(0) = 0. \end{cases}$

7(**10pts**) Fill in the following blanks and justify your answers:

(a) If $\mathcal{L}\{f(t)\}(s) = \dfrac{s}{(\sqrt{s}+1)^3}$, then $\mathcal{L}\{e^{-2t}f(t)\} =$ __________.

(b) If $\mathcal{L}\{f(t)\}(s) = \dfrac{s}{(\sqrt{s}+1)^3}$, then $\mathcal{L}\{u(t-2)f(t-2)\} =$ __________.

(c) If $\mathcal{L}^{-1}\{F(s)\}(t) = \dfrac{t+5}{t^2+1}$, then $\mathcal{L}^{-1}\{e^{-5s}F(s)\}(t) =$ __________.

8(**15pts**) Use the Laplace transformation method to solve this system of equations: $\begin{cases} x' = x + y \\ y' = -x + 1 \\ x(0) = 0, \ y(0) = 0. \end{cases}$

The End