Math 221 Test 3 **Fall 2011**

Name:______________________________ Score:__________

Instructions: You must show supporting work to receive full and partial credits. No text book, personal notes, formula sheets allowed. One formula sheet will be provided.

1(**15pts**) For this system of equations

$$\begin{cases} x' = y - x^2 \\ y' = 2x - y \end{cases}$$

(a) Verify that $(2, 4)$ is an equilibrium point for the system.

(b) Use linearization to determine the stability type of this equilibrium point.

2(**15pts**) Sketch a phase portrait for the system of two competing species in the first quadrant only

$$\begin{cases} x' = x(1 - 3x - 2y) \\ y' = y(1 - y - x) \end{cases}$$

Make sure to include the following: both x- and y-nullcline, typical vector fields on and off the nullclines, equilibrium points, and typical solution curves. Describe the long term behaviors of all solutions, and their ecological interpretations.

3(**15pts**) Use definition *only* to find the Laplace transform of the function $f(t) = e^{2t}$. Using any other method receives no credit.

4(**20pts**) Find the Laplace transforms of these functions

(a) $(t + \sin t)(e^{2t} + 1)$

(b) $f(t) = \begin{cases} \sin t, & t < 2 \\ 1, & 2 \leq t \end{cases}$ which must be first expressed in terms of unit step functions.

5(**20pts**) (a) Find the Laplace inverse of $\dfrac{3s^2 + 7s + 14}{(s-1)(s^2 + 4s + 7)}$

(b) If $\mathcal{L}^{-1}\{F(s)\}(t) = \dfrac{\sin t}{t^2 + 1}$, find $\mathcal{L}^{-1}\{e^{-2s}F(s)\}(t)$.

6(**15pts**) Use the Laplace transformation method to solve this system of equations:

$$\begin{cases} x' = 2x + y \\ y' = -x + \delta(t - 2) \\ x(0) = 0, \ y(0) = 0. \end{cases}$$

Bonus 2pts: Newtonian mechanics was discovered in (a) 1680s, (b) 1780s, (c) 1880s, (d) none of the above.

The End