Math 221 Final Exam **Fall 2006**

Name:________________________ Score:__________

Instructions: You must show supporting work to receive full and partial credits. No text book, personal notes, formula sheets allowed. One formula sheet will be provided.

1(**20pts**) (a) If $u(0.1) = 1$ and $u'(0.1) = 0.5$, what is $u(0.2)$ according to Euler's method with a time increment $h = 0.1$?

(b) Sketch a phase line of the equation $u' = (u^2 - 4)(u + 1)^2$ and classify the stability of the equilibrium points.

2(**20pts**) (a) Find a general solution to the separable equation $u' = u^2 + xu^2$.

(b) Find a general solution to the linear equation $u' + 2xu = x^3$. (Note: $\int te^t dt = te^t - e^t + C$.)

3(**20pts**) (a) Find a general solution to the equation $u''' + 4u'' + 4u' = 0$.

(b) Find a **FORM** only for the nonhomogeneous equation $u''' + 4u'' + 4u' = x + 2e^{-2x}$. (**Do not solve for the coefficients.**)

4(**20pts**) (a) Verify that $u_1(t) = t$ is a solution to the equation $u'' - \dfrac{t+2}{t}u' + \dfrac{t+2}{t^2}u = 0$.

(b) Find a second solution of the form $u_2(t) = tv(t)$, i.e., determine $v(t)$.

5(**20pts**) (a) Verify that $u_1(x) = \frac{1}{x}, u_2(x) = x$ are solutions to the homogeneous equation $x^2u'' + xu' - u = 0$.

(b) Use the method of variation of parameters to find a particular solution of $x^2u'' + xu' - u = \sqrt{x}$.

6(**20pts**) Consider the system of equations $\begin{cases} x' = -x - 2y \\ y' = 4x - 5y \end{cases}$

(a) Find a general solution of the system.

(b) Sketch a phase portrait of the system and classify the stability of the equilibrium point $(0, 0)$.

7(**20pts**) (a) Use the **Convolution Formula** to find the inverse Laplace transform of $\dfrac{1}{s(s^2+4)}$.

(b) Solve the initial value problem $u'(t) + 4u(t) = \delta_3(t),\ u(0) = 0$.

8(**20pts**) Use the Laplace Transform method to solve $\begin{cases} x' = -y + f(t) \\ y' = x \\ x(0) = 0,\ y(0) = 0, \end{cases}$ where $f(t) = \begin{cases} 1, & t < 2 \\ 0, & t \geq 2 \end{cases}$

9(**20pts**) Use Euler's method to approximate the solution to the initial value problem $\begin{cases} x' = y \\ y' = (x^2 - 1)y - x \\ x(0) = 1,\ y(0) = 0, \end{cases}$ at $t = 0.5$ in 2 time increment steps.

10(**20pts**) Use the method of phase plane to sketch a phase portrait of the system of equations $\begin{cases} x' = -2x + 3y \\ y' = 4y \end{cases}$

Make sure to include all essential features (e.g. both x- and y-nullclines, equilibrium points, vector fields both on and off the nullclines, special orbits that come out or go into the equilibrium points if any, and typical orbits). Classify the equilibrium points as sink, source, saddle, or spiral sink, source, center.

Bonus 2pts: If I ace this class, it is because I ________________

The End