Math 221 Test 2 **Fall 2004**

Name:______________________________ Score:________________________

Instructions: You must show supporting work to receive full and partial credits. No text book, notes, formula sheets allowed.

1(**18pts**) Consider the differential equation $\frac{dy}{dt} = (1-y^2)(y-2)^2$.

(a) Sketch the graph of the $\frac{dy}{dt}$ v.s. y. (Use your calculator.)

(b) Sketch the phase line.

(c) Find the equilibrium points and classify their stabilities.

(d) Sketch a few typical solutions $y(t)$ v.s. the independent variable t.

(e) Describe the long time behavior of the solution with any initial condition satisfying $y(0) > 3$.

2(**18pts**) Consider the equations

$$\begin{cases} x' = xy \\ y' = (1+y^2)(1-x) \end{cases}$$

(a) Derive the equation for its trajectories in the xy-plane.

(b) Solve the equation for the trajectories.

(c) Sketch the trajectory that satisfies $x(0) = \sqrt{2}, y(0) = 1$. (Use your calculator.)

3(**18pts**) Consider the system of two competing species

$$\begin{cases} x' = x(1-x-ay) \\ y' = y(1-y-x). \end{cases}$$

(a) Sketch 2 phase portraits, one for the case $a > 1$, the other for the case $a < 1$. (Make sure to include nullclines, typical velocity vectors on and off the nullclines, and typical trajectories.)

(b) Describe the long term behaviors of the solutions in both cases. Highlight the main differences between them.

4(**18pts**) (a) Find the solution to the initial value problem $x'' + 5x' + 4x = 0, x(0) = 1, x'(0) = 0$.

(b) If the roots of the characteristic equation of the homogeneous equation $ax^{(4)} + bx''' + cx'' + dx' + ex = 0$ of real, constant coefficients are -2 and $-2 \pm i$. Find the general solution to the equation.

5(**18pts**) (a) Use the method of undetermined coefficient to find a particular solution to the equation $x'' + 2x + 1 = 1 + 2t$.

(b) If the roots of the characteristic equation of the homogeneous equation $ax^{(4)} + bx''' + cx'' + dx' + ex = 0$ are $-1, -2, \pm 3i$, find the FORM of a particular solution to the nonhomogeneous equation $ax^{(4)} + bx''' + cx'' + dx' + ex = g(t)$ for 2 cases respectively: (1) $g(t) = t\cos 3t$, and (2) $g(t) = 3e^{-2}\sin 3t$.

6(**10pts**) A 4-kg mass stretches a spring hanging from the ceiling by 50 cm. If the damping constant for the system is 5 N-sec/m, determine the steady-state for the mass after an external force of $F(t) = 2\sin t$ N is applied to the system.

END