Math 221/821 — Final Exam — Dec. 14, 1999

Name:______________________________ **Score:**____________

Instructions: You must show supporting work to receive full and partial credits.

1(16 pts) Find the general solution to the separable equation $\dfrac{1}{\cos x}\dfrac{dy}{dx} = \dfrac{1}{(y+1)}$.

2(16 pts) Solve the initial value problem of the linear equation $y' + \frac{2}{t}y = 1, y(1) = 2$.

3(18 pts) (a) Use the Euler method with step size $h = 0.2$ to approximate the solution to the initial value problem $y' = 2t - y^2, y(0) = 0$ over the interval $0 \le t \le 0.4$.
(b) Sketch your approximating solution.

4(15 pts) Find the general solution to the equation $y'' + 2y' + y = 0$.

5(16 pts) Use the method of undetermined coefficients to find a suitable **form** for a particular solution $Y(t)$ to the equation

$$y' - y = t^2 - (t-1)\sin 3t + 2e^t.$$

(DO NOT SOLVE FOR $Y(t)$ BY FINDING ALL THE UNDETERMINED COEFFICIENTS!)

6(15 pts) Use the method of undetermined coefficients to find a particular solution to the equation $y' - 3y = 3t$.

7(16 pts) (a) The eigenvalues and their eigenvectors for a linear equation $\vec{x}' = A\vec{x}$ are given as $\lambda_{1,2} = 1 \pm i2$ and $\vec{u}_{1,2} = \begin{pmatrix} 1 \mp i \\ 2 \end{pmatrix}$, respectively. Find the general solution to the equation.
(b) The eigenvalue of a linear equation $\vec{x}' = A\vec{x}$ is $\lambda = -2$ and it is repeating. An eigenvalue $\vec{u}$ and a solution to the equation $(A - \lambda I)\vec{v} = \vec{u}$ are given as $\vec{u} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$, $\vec{v} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$, respectively. Find two linearly independent solutions to the equations and sketch a phase portrait for its trajectories.

8(18 pts) (a) Find the eigenvalues of the matrix $\begin{pmatrix} 3 & -5 \\ 2 & -4 \end{pmatrix}$.
(b) Find an eigenvector for the largest eigenvalue.

9(18 pts) A partial phase portrait is sketched for a system of competing species.
(a) Completing the phase portrait by marking the equilibrium points, some representative field vectors on the nullclines and the regions bounded by the nullclines, some representative solution trajectories.
(b) Under what conditions do they co-exist? Under what conditions do they not?

10(18 pts) Find the Inverse Laplace Transforms of the given functions
(a) $\mathcal{L}^{-1}\left[\dfrac{1}{s^2 - 2s - 3}\right](t)$. (*Hint:* Use partial fraction first.)
(b) $\mathcal{L}^{-1}\left[\dfrac{1}{s^2 - 2s + 3}\right](t)$. (*Hint:* Use complete squaring first.)

11(16 pts) Find the Laplace Transform $\mathcal{L}[y(t)](s)$ of the solution $y(t)$ to the initial value problem

$$y'''(t) - 2y(t) = 2e^{2t}\cos 3t, \quad y(0) = 2,\ y'(0) = 0,\ y''(0) = 0.$$

(DO NOT SOLVE FOR $y(t)$!)

12(18 pts) Use the method of Laplace Transformation to find the solution $y(t)$ to the initial value problem

$$y'(t) - y(t) = 1, \quad y(0) = 2.$$

END