Math 221 Test 3 **Fall 2003**

Name:_____ Score:_____

Instructions: You must show supporting work to receive full and partial credits. No text book, personal notes, formula sheets allowed. One formula sheet will be provided.

1(**10pts**) Use the method of elimination to find a general solution to the system of equations

$$\begin{cases} x' = -4x + 6y + t \\ y' = 5y - 3x \end{cases}$$

2(**15pts**) Consider the equations

$$\begin{cases} x' = x - y + 2 \\ y' = x + y \end{cases}$$

(a) Find the equilibrium point of the system.

(b) Sketch the x-nullcline and y-nullcline, and a few representative vector fields on the nullclines and in the regions partitioned by the nullclines.

(c) Sketch a few typical solution curves to complete a phase portrait of the system.

3(**10pts**) Use definition *only* to find the Laplace transform of the function

$$f(t) = \begin{cases} e^{-t}, & t < 2 \\ 0, & t \geq 2 \end{cases}$$

Using any other method receives no credit.

4(**20pts**) Find the Laplace transforms of these functions

(a) $(\sin t - 2t)^2$

(b) $f(t) = \begin{cases} t, & t < 2 \\ 1, & 2 \leq t < 3, \\ 0, & t \geq 3 \end{cases}$ which must be first expressed in terms of unit step functions.

5(**20pts**) Find the Laplace inverses of these functions

(a) $\dfrac{(s+1)e^{-3s}}{s^2 + 4s + 8}$

(b) $\dfrac{s^2 - 5s + 3}{(s+1)(s^2 - 4s + 4)}$

6(**15pts**) Solve the initial value problem: $\begin{cases} x'' + 2x' + 2x = 2\delta(t-2) \\ x(0) = 1,\ x'(0) = 0. \end{cases}$

7(**10pts**) Fill in the following blanks and justify your answers:

(a) If $\mathcal{L}\{f(t)\}(s) = \dfrac{s}{(\sqrt{s}+1)^3}$, then $\mathcal{L}\{e^{-2t}f(t)\} =$ _____.

(b) If $\mathcal{L}^{-1}\{F(s)\}(t) = \dfrac{t+5}{t^2+1}$, then $\mathcal{L}^{-1}\{e^{-5s}F(s)\}(t) =$ _____.

Bonus 5pts: Use the Laplace transformation method to solve this system of equations: $\begin{cases} x' = x + y \\ y' = -x + 1 \\ x(0) = 0,\ y(0) = 0. \end{cases}$

Warning : No partial credit will be given. Don't try if you don't have time to waste.

The End