Instructions: You must show supporting work to receive full and partial credits. No text book, notes, formula sheets allowed.

1 (15pts) (a) Verify whether or not \( y(x) = -\cos(2x) \) is a solution to the equation: \( y'' + xy = 2\sin(2x) - x\cos(2x) \).

(b) Does \( x^2 - \sin(x + y) = 1 \) define an implicitly solution to the equation \( \frac{dy}{dx} = 2x\sec(x + y) - 1 \)?

2 (10pts) (a) The isoclines: \( f(t, x) = -1, -0.5, 0, 0.5, 1 \) of a differential equation \( x' = f(t, x) \) is given in figure (a). Sketch the slope field and the solution starting approximately at \((-5, 1)\).

(b) The vector field of an equation is given in figure (b). Sketch solutions that go through these points: (i) \((-3, 1)\), (ii) \((0, 0)\).

3 (15pts) Use Euler’s method to approximate the solution to the IVP: \( y' = x^2 - y^2 \), \( y(0) = 1 \) in the interval \([0, 1]\) by discretizing the interval into 4 equal parts. Sketch your approximating solution.

4 (15pts) Find the solution to the initial value problem \( \frac{dx}{dt} = t + t^2x \), \( x(0) = 1 \).

5 (15pts) Find a general solution to the linear equation \( \frac{dx}{dt} + xt = t^2 \).

6 (10pts) A brine solution containing 0.2 lb of salt per gallon is used to salinize a play pool containing 500 gallon of pure water. If the brine solution flows into the pool at 5 gallons per minute, and the well mixed solution flows out at the same rate, set up a differential equation for the amount of salt in the pool at any time. How much salt is in the pool 10 minutes after the solution starts to flow?

7 (10pts) During the summer the temperature inside a van is 28°C after the driver turns off the air conditioner. If the temperature outside is 37°C and it takes 1 minute for the van to heat up to 30°C, write a differential equation for the temperature and estimate how many minutes will it take for the van to reach 35°C inside.

8 (10pts) A parachutist weighing 75 kg steps out an airplane 3000 m above the ground. Assume the air resistance is proportional to the velocity with which she is falling and the proportionality is \( b = 20\text{N-sec/m} \), write a differential equation for the falling parachutist and find her falling velocity 1 minutes later.

END