Manual Test 2, Sel. Key, Spring '01

1 (15pts): \[ y'' + ay' + by = 0, \ r^2 + ar + b = 0, \ \Gamma_1, \ \Gamma_2 = \frac{-a \pm \sqrt{a^2 - 4b}}{2} = -2 \pm i; \]
\[ y(t) = C_1 e^{-2t} \cos t + C_2 e^{-2t} \sin t, \ -2 = y(0) = C_1, \ 1 = y'(0) = -2C_1 + C_2 \]
\[ \rightarrow C_1 = -2, \ C_2 = 1 \neq 2, -3, -3 \]
\[ (y(t)) = -2e^{-2t} \cos t - 3e^{-2t} \sin t \]

2 (20pts): \[ y'' + 2y' + y = 0, \ r^2 + 2r + 1 = 0, \ r_1 = -2 \pm i; \]
\[ y(0) = e^{0t}, \ y_2 = e^{0t} \]

(a) \[ g(t) = 1 + t, \ y_p(t) = t^2 [A + Bt] \]
\[ \Rightarrow y(t) = A + Bt \]

(b) \[ g(t) = e^{t} \sin t, \ y_p(t) = t^2 [Ae^{t} \sin t + Be^{t} \cos t] \]
\[ \Rightarrow y(t) = Ate^{t} \sin t + Bte^{t} \cos t \]

(c) \[ g(t) = e^{t}, \ y_p(t) = e^{t} \]
\[ \Rightarrow y(t) = y_1(t) + y_2(t), \ y_2(t) = e^{t} \]

3 (25pts): \[ y'' - ay = t^2, \ y'' - ay = 0, \ r^2 - 1 = (r + 1)(r - 1) = 0, \ r_1 = -1, \ r_2 = 1, \ y_1 = e^{-t}, \ y_2 = e^{t} \]

(a) \[ g(t) = t^2, \ y_p(t) = t^2 [A + Bt + Ct^2] \]
\[ \Rightarrow y(t) = B + 2ct, \ y_p'' = 2c \cdot 2c \cdot 2c \]
\[ \Rightarrow y_p'' = 2c(A + Bt + Ct^2) = t^2 \]}
\[ \Rightarrow c = 1, -B = 0, 2c = A \]
\[ \Rightarrow A = 2c = -2 \]
\[ \Rightarrow y_p = -2 - t^2, \ y(t) = C_1 e^{t} + C_2 e^{t} \]
\[ \Rightarrow y(t) = C_1 e^{t} + C_2 e^{-t} - 2 - t^2 \]

4 (15pts): \[ \frac{y(t)}{3} = C_1 + C_2 t + C_3 e^{3t} + C_4 e^{-3t} \cos t + C_5 e^{-3t} \sin t \]

5 (25pts): \[ y'' - 2y' + y = \frac{e^t}{t}, \ y_1(t) = e^t, \ y_2(t) = te^t \]
\[ W[y_1, y_2] = \left| \begin{array}{c} e^t & te^t \\ e^t & e^t + te^t \end{array} \right| = e^{2t} \]
\[ y(t) = -\frac{\int y_1 g \, dt}{W[y_1, y_2]} + \frac{\int y_2 g \, dt}{W[y_1, y_2]} \]
\[ = -\int dt = -t, \ y_p(t) = \int \frac{y_1 g}{W[y_1, y_2]} \, dt = \int \frac{e^t \cdot e^t}{e^{2t}} \, dt = \int e^{-t} \, dt = e^{-t} \]
\[ \Rightarrow y_p(t) = y_1(t) + y_2(t) = -te^t + te^{t} \ln t, \ y_p(t) = te^{t} \ln t \]