Math 221/821 **Test 3** **April 19, 2001**

Name:_____ Score:_____

Instructions: You must show supporting work to receive full and partial credits. No text book, notes, formula sheets allowed.

1(25 pts) (a) Is the system of equations

$$\dot{x} = x(1 - 2x - y), \quad \dot{y} = y(1 - y - x)$$

a model for two competing spieces? or two cooperating spieces? or a prey and a predator?

(b) Sketch a phase portrait for the system which must include: the nullclines, the equilibrium points, vector field at typical points on the nullclines and in the regions bounded by them, and solutions curves illustrating typical behaviours of the interaction.

(c) Describe in a few sentences typical outcomes of the interaction.

2(10 pts) Use the definition to find the Laplace transform $\mathcal{L}\{f(t)\}(s)$ at $s = 5$ for

$$f(t) = \begin{cases} e^{-2t}, & 0 \le t < 3 \\ 0, & 3 \le t < \infty. \end{cases}$$

3(15 pts) Find the Laplace transforms $\mathcal{L}\{f(t)\}(s)$ for the following functions

(a) $(t+1)^2$

(b) $e^{-t}\cos(3t) - 2te^{5t}$

(c) $u(t-2)g(t)$ if $\mathcal{L}\{g(t+2)\}(s) = \dfrac{1}{\sqrt{s^2+4}}$.

4(20 pts) Find the inverse Laplace transforms $\mathcal{L}^{-1}\{F(s)\}(t)$ for the following functions

(a) $\dfrac{s}{2(s-3)^2+2}$

(b) $\dfrac{1}{s^3} - \dfrac{1}{s^2+2}$

(c) $\dfrac{5}{s^2+4s+8}$.

5(15 pts) (a) Find the partial fraction **form** for $\dfrac{2s+1}{s^4+2s^3+2s^2}$. **Do not solve for the partial fraction constants.**

(b) Write the function $f(t)$ in terms of linear combinations of some unit step functions $u(t-c)$ and then find the Laplace transform $\mathcal{L}\{f(t)\}(s)$

$$f(t) = \begin{cases} 2, & 0 \le t < 1 \\ 0, & 1 \le t < 3 \\ 1, & 3 \le t < \infty. \end{cases}$$

6(15 pts) Use the Laplace transformation method to solve the initial value problem

$$y' - y = u(t-2), y(0) = 0.$$

(The answer is $u(t-2)(-1+e^{t-2})$.)

Bonus(3 pts) Who is UNL's new Chancellor? and where is he from?

END